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STATISTICS FOR BEGINNERS IN EDUCATION

STATISTICS FOR BEGINNERS IN EDUCATION

AN ELEMENTARY PRESENTATION AND EXPLANATION OF SIXTY-TWO TERMS IN EDUCATIONAL STATISTICS

BY

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TO

PERNA AND ANNETTE

TWO LOYAL SISTERS

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PREFACE

One reason why education is not yet an exact science is found in the wavering terminology in use. It is true that in the measurement of length a foot is not always the same because environmental influences such as temperature and the differing customs of nationalities tend to make tools of measurement unreliable. But in education our units of measurement are still more indefinite as to quantity. What is a "grade"? What is "success" in fifth-grade arithmetic?

Furthermore, the novice entering any profession is unfamiliar with that part of the vocabulary used which is peculiar to that field of endeavor. This is particularly true in a realm in which a science is in the making. It is true in education because education is coming rather rapidly to have a language of its own. One measure of the successful introduction of the prospective teacher into the field of education will be found in the facility with which he becomes acquainted with the new words found there and gains true insight into their real meaning.

The young high-school graduate, enrolling for the first year's work in the education and educational psychology courses of a teacher-training institution is faced with this concrete difficulty. He is not sure what it is all about because he does not understand fully the peculiar meaning evidently attached to old familiar words in this new work, nor is he able to grasp

early enough the significance of the strange terminology coming to his attention every day in classroom discussion and in reading. A training teacher said the other day that her student teachers often complained that they did not know what such words as "interest" or "discipline" really meant now. And, as for terms such foreign as "median" or "deviation," they were all at sea.

This book is an attempt to meet the need described above, so far as the simpler phraseology of educational statistics is concerned. It does not attempt to give the student understanding of the entire vocabulary of education, but it does aim to introduce him to those quantitative terms, those "terms of amount," so often occurring in classroom conversation and in required reading. Of course, no attempt is made to include in the exposition all the significant words of educational statistics. Only those most commonly found are dealt with. What to include has been determined by analyses of the statistical terminology used in a representative sampling of current educational journals read by both high-school [1] and elementary-school [2] teachers.

The aim of this treatise is not to develop skill (1) in the derivation of formulae and (2) in the use of the statistical method in education. The latter might be the objective in a second, or introductory, course; and the former in a third, advanced attack on statistical prob-

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[1] J. L. La Poe, "The Senior High-School Principals' Professional Magazines," *Educational Research Bulletin,* Vol. 6, pp. 259-261 (September 14, 1927).

[2] E. Lowell Kelly, and F. L. Whitney, "Educational Magazines Read by 500 Elementary-School Principals and Classroom Teachers," *Elementary School Journal,* Vol. 29, pp. 176-180.

lems. What is attempted is to state simply and illustrate clearly in the field of education the meaning of the statistical terms included in the book.

It is believed that a treatment of the problem of obtaining a pre-professional vocabulary, such as is offered in this little book, will find a place in the professional improvement of many educators in many situations at many stages of advancement. Many teachers, administrators, and supervisors in service have not come into contact as yet in the higher institutions of learning with courses which will give them a command of the new words of educational statistics. But they must know a respectable number of them in order to read professional literature with understanding. This book can be used in local study groups or in connection with state reading circle requirements with such a purpose in view. Normal classes in the secondary field need a simple textbook fitted to their capabilities and to the limited time allotments in a crowded school year. Most serious of all, that large group of tyros in education, over 200,000 strong, coming for the first time into contact with the literature of a chosen occupation, suffer an insurmountable handicap unless some definite, organized plan is made to aid them to an early grasp of the concepts of the new undertaking.

Ordinarily, there is not time in the crowded program of the freshman year of the typical teacher-training institution to require a four-hour course in educational statistics, even if it would be desirable. But this book has been written with the viewpoint that such a course is not necessary at that time. Two hours very probably would be a maximum time assignment, if it be

thought desirable to present the content of the textbook in a separate course. And very possibly one hour would be all that would be needed. It might be better, too, to use the book in connection with and as an early part of the first courses in education or educational psychology, requiring its reading and giving such time to class demonstration, discussion, and testing as seemed necessary. The object would be to get a "reference knowledge" of its content, and at the same time to establish a rather definite halo of meaning in connection with each of the sixty-two statistical terms discussed.

There is, of course, nothing original in this book but the basic thesis—that first of all the educational neophyte needs, not skill in statistical technic, but a background of word conception sufficient to understand references to statistical terms which appear in the commonly read journals and books of education.

Acknowledgment is made to Dr. C. H. Judd, Dr. L. D. Coffman, and Dr. H. F. Clark for courteous permission to use as illustrative material certain data from their publications.

I appreciate very much the interest and help of the Colorado State Teachers College faculty during the writing of the book, in particular that of President George Willard Frasier and members of the departments of educational psychology and education.

I wish to acknowledge the good help of Miss Jessie L. Thompson, Research Secretary, in the preparation of the manuscript and of Philip M. Condit, Research Fellow, in careful reading of the details of tables and figures.

F. L. W.

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TO THE INSTRUCTOR

If you have read the preface, you will understand the general purpose of this textbook. The details of its use by the student are suggested to him as the exposition proceeds. These details are, of course, but suggestive to you. Your judgment and the conditions under which you are using the book may require quite different technics.

Do you not agree that the important objective which should be in mind will be to furnish each student with a word conscience, so tender that he will hesitate long enough when a new term appears in his reading to get from any quickly available source some idea of its meaning as used? If the student as future teacher can be started out in his preparatory experience with this problem attitude toward the situations he is studying, there will be more likelihood of his pursuing his teachers-college courses on a higher level of understanding and of his performing as a teacher in service with more success.

Material for "Illustrative Exercises" has been included in Chapters II to VII inclusive. The suggestion is that these problems or others of a similar nature be used as needed in the classroom for demonstration by the instructor, the class following each step in the calculation with question and discussion. This may help to fix the meaning of the terms derived.

STATISTICS FOR BEGINNERS IN EDUCATION

CHAPTER I

HOW MUCH DO I KNOW ABOUT EDUCATIONAL STATISTICS?

Mason was a country town of less than 10,000 people located in one of the central Mississippi Valley states. The public schools were of the ordinary sort. The superintendent was a graduate of the state university, but had not taken any education courses there. His ideals of teaching were based on his memory of his own childhood experience in a similar school where conditions had been, as in Mason, ruled by tradition and "what has been." There were about 1,200 pupils in the public schools in classes ranging from grades one to twelve inclusive, the 8-4 plan. They came from middle-class homes. There was no wealth and no poverty, except that Colonel Brush, the banker, was reported to be worth at least $50,000 and Sore-eye Truesdell, the town nit-wit, spent frequent lengthy visits at the county "poor farm."

About five years ago there was a change in the school superintendency in Mason. The interest and energy of the superintendent, Mr. Blank, had been

gradually drifting toward business, until finally he decided to shift all of his time and attention from school work, instead of 80 per cent of it. The board of education wrote to the state university for another man, and this time they were lucky. Mr. White was recommended, and after a personal visit he was elected for a three-year term, but at a salary which half of the board thought scandalously big for a "teacher" and which all of them thought best not to advertise.

Mr. White had begun his work in the public schools as a country school teacher, serving during a "winter term" of about two months and a half and during the warmer season attending the state agricultural college. Later he was graduated from the four-year course at the state teachers college. After acting as principal and superintendent in two small village schools, he spent a year in the college of education at the state university, earning his master's degree. Here he came into contact with one of the outstanding professional leaders in American education, and the beginnings of education as a science were revealed to him. When he was elected at Mason, he decided to put into practice the principles and methods with which he had become familiar and which he had studied and illustrated in his master's investigation. This was a survey of a group of small school systems such as those in which he had been serving.

But Mr. White was possessed of a good supply of

native "common sense" and social intelligence. He had seen one of his former fellow students at the university go on the rocks in his first superintendency as a "measurement hound," after attempting to reclassify the whole school on the basis of one group test in intelligence. He decided to go slowly.

The teaching force of about three dozen was typical of systems in that locality: A few "home girls," more or less competent, but more less competent than more so; eight or ten "old, experienced" grade teachers; the majority of the high-school teachers young, inexperienced, and without adequate professional training. But the corps included a very few exceptional people, in particular two whom the superintendent had brought from the college of education at the university, teachers thoroughly imbued with the coming scientific spirit in education and possessed of detailed knowledge and skill in the use of the newer tools and methods.

The new superintendent, at his first general teachers' meeting, referred briefly to the newer phases of modern education and specifically to the measurement movement which at that time had been in progress for about ten years. In particular, he emphasized the fact that for many years in American education the problem had been that of handling public school children in groups, but teaching them in accordance with their individual needs. The substance of his remarks on this theme was as follows:

Teaching was at first an individual matter, the contact of one child mind with one adult mind. It took the form of tutoring, or of parent or priest instruction. When it was necessary for a group to be present, teaching was still individual. One child at a time was called up for treatment, the others lying fallow meanwhile. This type of management persisted in a few isolated places even down to the second decade of the last century. It was found notably in Scotland then.

Schooling by this method was costly and for that reason exclusive. The education of all of the people was not possible. A very few prospective leaders only could benefit from it.

At the close of the seventeenth century (1684), class instruction was first employed. The Christian Brethren in France, in their organization for a plan more democratic than the tutorial system, went about two by two and taught the children in groups.

Obviously, group teaching is cheaper; and largely because of this, schooling has been by group instruction ever since. This organization of activities reached its ultimate and logical form in the monitorial schools of Lancaster which were introduced into the United States near the beginning of the last century and attained popularity in many of the larger eastern cities. By this plan, the unit cost of instruction was reduced to a very low point by enlarging to unbelievable numbers the pupil group. The teacher's duties were largely managerial. His function was simply "to organize, reward, punish, and inspire," and many very cheap fractions of teachers (one for every ten pupils), the monitors, did the teaching.

This furor of economy ran its course. Its many fallacies became apparent through experience. Gradually the size of the pupil group was reduced to the number who might be seated in a room of ordinary size. The plan for

pupil participation was changed materially or dropped entirely. There are many interesting exceptions to the rule, but the typical classroom situation today finds one teacher facing 30 or 40 children in a room of their own.

The problem ever since class instruction was adopted has been, How to teach *en masse* and yet reach the individual child effectively? This problem was held rather unconsciously until quite recently. Dependence was placed upon what has been called "blunder-buss teaching." Aim was taken at that mythical entity, "the average child"; and to the flaring bell muzzle of the teaching method was left the responsibility for hitting the entire group.

It has been but recently that this problem has been entirely articulate, that teachers have said, "Here is my group, but how can I reach each individual child to inspire him to effort commensurate with his own ability? Here are 40 children, but I must know how to arrange matters so that each individual may be permitted to progress at his own rate through the use of subject matter entirely fitted to his interests, capabilities, and after-school activities."

As soon as teachers began to study the group in terms of the individual and the individual in terms of his varying equipment and abilities, many interesting situations were discovered, many groups of facts which threw much light on what was desirable and possible in the ordinary school situation for the benefit of the individual child.

At the close of the teachers' meeting, Mr. White emphasized the importance of professional improvement on the part of every teacher under contract and recommended that as many as so desired read Thorndike's little book on *Individuality*,[1] a dozen

[1] E. L. Thorndike, *Individuality*, Riverside Educational Monograph (Houghton Mifflin Co., 1911).

copies of which they would find in his office. Further, he called for volunteers to meet him on Saturday morning to decide on some study which might benefit the school as a whole.

A group of a half dozen appeared, the high-school principal who no doubt did not dare to remain away, the two new teachers from the university, and three elementary teachers who the superintendent soon discovered were outstanding in professional attitude and teaching ability. After discussion, it was decided that during the first semester of the year it would be most valuable to make a checking of the school situation in the district to discover real conditions as a basis for future improvement, to make a "difficulty analysis" for diagnostic purposes. A number had read the book mentioned at the teachers' meeting three days before, and it was the consensus that first of all a checking of differences which might exist among the pupils should be made, considering (*a*) school and grade groups, (*b*) individual children, and (*c*) specific abilities within the child himself.

In planning, a start was made with eight hypothetical theses:

I. Differences among groups of school children
   1. School children of the same grade, but seated in different classrooms, differ in their reaction in general to the total school situation.
   2. School children react differently to a test of specific abilities both among grade groups and

among larger groups, including large portions of school systems.

II. Differences among individual school children
   3. School children seated in the same room and grade differ in the stage of physical maturity reached.
   4. School children classified in the same grade differ in age, and those of the same age are doing work in different grades.
   5. School children in the same grade differ in their reaction to school subjects.
   6. School children of similar life age differ in their mental development.

III. Differing abilities within the individual child
   7. Chronological age and mental age differ in many school children.
   8. Abilities and groups of abilities differ in many school children.

I. GENERAL DIFFERENCES

Miss Green, one of the new teachers from the university, was made chairman of the voluntary research committee. She was a high-school teacher, but had taught in the elementary school before entering secondary work. After the meeting, in a conversation with the principal of the large elementary school of the system, the first tentative generalization (I, 1) that had been made came under discussion.

School children of the same grade, but seated in different classrooms, differ in their reaction in general to the total school situation.

Miss Poe, the principal, said, "Yes, that is true in many cases. There are the sixth-grade classes in my building and in that across the tracks. They are as different as can be. My Miss Doe certainly has a fine room this year. I can remember those children a number of years back. Miss Blank had them when I began here, and she gave them a good start and promoted nearly all of them. And then Miss Gray had them, and she kept them together and progressing. And now Miss Doe has them, and they are a credit to the school and to their teacher. But that sixth-grade room in the ward building is a fright. Miss Doe was comparing their passing marks with hers last week. Her median was 88.5, and Miss Roe's was 68.0. Here are a table and a graph that show up the two classes in great shape. You can't expect a teacher to do much with a class like Miss Roe's." And Miss Poe handed Miss Green Table I and Figure 1.

"Well, that seems to prove the first thesis our committee wrote to-day—at least for those two classes," said Miss Green. "We must see if such big contrasts are easy to find, and what the reasons are, and what to do about it. I wonder whether that slow class can't be made to do a little better?"

2. MORE SPECIFIC ABILITIES

As the committee continued their search for facts about individual differences among the 1,200 school

TABLE I

THE AVERAGE OF ALL "PASSING MARKS"[1] OF TWO SIXTH-GRADE CLASSES IN ALL OF THEIR SCHOOL SUBJECTS, MASON CITY SCHOOLS.

Class Interval (per cent)	Miss Doe's Class	Miss Roe's Class
1	2	3
95 — 99	2	
90 — 94	10	
85 — 89	24	
80 — 84	1	1
75 — 79	1	1
70 — 74		10
65 — 69		20
60 — 64		6
55 — 59		2
Total	38	40
Median	88.5	68

[1] The critical grade, the "passing mark," in this old-fashioned system was 75 per cent.

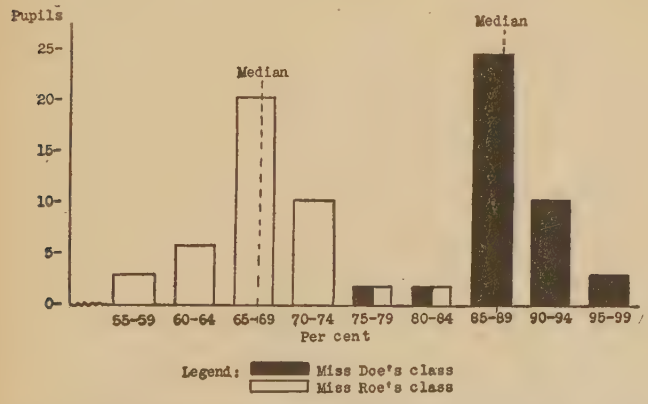

FIG. 1.—THE ARITHMETIC AVERAGE OF ALL THE MARKS OF TWO SIXTH GRADE CLASSES IN ALL OF THEIR SCHOOL SUBJECTS, MASON CITY SCHOOLS.

children in Mason, they came across many situations which threw light upon the eight points of inquiry with which they had started their work, and in particular illustrations of the second (I, 2) were not hard to find.

School children react differently to a test of specific abilities both among grade groups and among larger groups, including large portions of school systems.

The first general test the superintendent decided upon was in spelling. He gave Ayres' "short list" [1] of ten words for each grade in grades two to eight inclusive. This had been used in the Springfield survey,[2] and later in Salt Lake City [3] and other public school systems. The committee persuaded the county superintendent to give the test also in the country schools, and all data were worked up in Mr. White's office.

When the figures were tabulated and comparative data included from Salt Lake City, the results appeared as in Table II and Figure 2.

Upon studying this table and figure, the committee found: (*a*) that the fifth-grade rooms in

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[1] L. P. Ayres, *A Measuring Scale for Ability in Spelling* (Russell Sage Foundation, 1915).

[2] L. P. Ayres, *The Public Schools of Springfield, Illinois: An Educational Survey,* Bulletin E 137 (Russell Sage Foundation, 1913).

[3] E. P. Cubberley, *School Organization and Administration* (World Book Co., 1916).

TABLE II

THE SPELLING ACHIEVEMENT OF 1,200 SCHOOL CHILDREN IN MASON, 2,600 IN THE RURAL SCHOOLS OF CLASS COUNTY, AND 3,988 IN SALT LAKE CITY, AYRES' "SHORT LIST."

Grade	Mason [1]	Country Schools [1]	Salt Lake City [2]
1	2	3	4
2	32.5	65	
3	39.8	61	89.9
4	64.2	68	78.8
5	64.5	69	87.6
6	70.3	59	86.8
7	73.7	59	87.1
8	59.3	51	82.2
Mean	57.8	62	85.4

[1] F. L. Whitney, *Report of the Superintendent of Schools, 1916-17* (Board of Education, Huron, South Dakota, 1917).
[2] E. P. Cubberley, *School Organization and Administration* (World Book Co., 1916).

Mason, in the rural schools, and in Salt Lake City all did better on the test than the eighth-grade groups; (*b*) that the lowest grades in Class County and in Salt Lake City did better than the highest grades; (*c*) that grade two did poorest in Mason, grade eight in the country, and grade four in Salt Lake City, etc. In fact, it appeared that here was a confirmation of the first part of the second thesis, and that school children really did react differently among grade groups to a test of specific abilities.

Further examination of the data showed also: (*a*) that Mason had five of its grade groups and Class County all below Ayres' average of 70, while Salt Lake City had all above; (*b*) that the average spell-

ing ability in Mason was 58 per cent of accuracy, in the rural schools 62 per cent, and in Salt Lake City 85 per cent; (*c*) that all the children in Salt Lake City scored higher than all in the local tests;

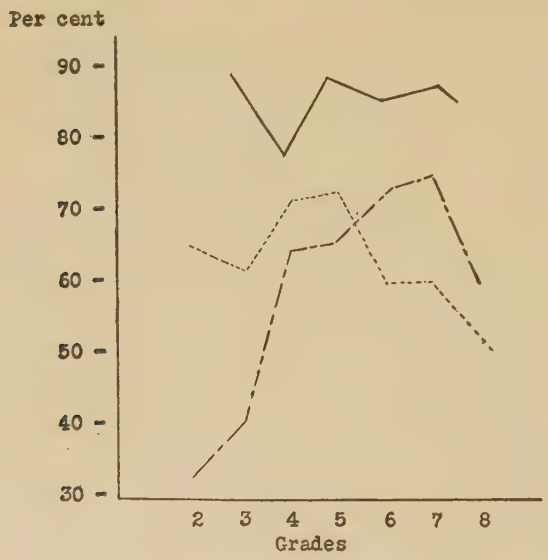

FIG. 2.—THE SPELLING ACHIEVEMENT OF 1,200 SCHOOL CHILDREN IN MASON, 2,600 IN THE RURAL SCHOOLS OF CLASS COUNTY, AND 3,988 IN SALT LAKE CITY, AYRES' "SHORT LIST."

(*d*) that the country pupils scored in general in terms of central tendency (mean) about 4 per cent higher than Mason's pupils; (*e*) that the total range in spelling scores in Mason was over 40 per cent,

in Class County 18 per cent, in Salt Lake City 11 per cent, etc. In fact, it appeared that here was illustration of the remainder of the second generalization (I, 2), and that school children did react differently to tests of specific abilities when large system groups are considered.

3. PHYSICAL DEVELOPMENT

The committee found many illustrations of differences among groups of school children, but in their opinion it would be more significant and would show greater evidence of a careful analysis of their teaching problems if they could discover teachers who were attempting to recognize differences in the capacities and capabilities of the individual pupils in their care. They made inquiry first of facts of physique which might illustrate their third hypothesis (II, 3).

School children seated in the same room and grade differ in the state of physical maturity reached.

An instance of this kind came to their attention almost immediately. The fourth-grade teacher in the central building had on her blackboard a chart the facts of which are shown in Table III and Figure 3. This small group of children, about nine years old, were measuring themselves every month and then drawing the height curve for the class together with the average line. They were all normal children

TABLE III

THE HEIGHT OF A SMALL FOURTH-GRADE CLASS AS MEASURED BY THEMSELVES IN ONE OF THEIR PERIODIC CHECKINGS OF PHYSICAL DEVELOPMENT.

Rank	Pupils	Height (inches)
1	2	3
1	Grant	58.0
3	Arthur	55.0
3	Donald	55.0
3	Virginia	55.0
5	Karl	54.5
8.5	Romayne	53.5
8.5	Eleanor	53.5
8.5	Jeanne	53.5
8.5	Helen	53.5
8.5	John	53.5
8.5	Adade	53.5
12	Wayne	53.0
13.5	Marvin	52.0
13.5	Harvey	52.0
15	Violet	51.0
16	Harriette	50.5
17	Elizabeth	50.0
	Q_3	54.8
	Median	53.5
	Q_1	52.0
	Q^1	1.4
	Range	8.0

[1] Quartile deviation.

from families rather above the average as to physical inheritance, nutrition, care, etc. But, even in so small a group, great individual differences appeared. There was a total range of eight inches, and among the middle half the range was nearly three inches. Grant

was as tall as his teacher, and when Harriette and Elizabeth were seated their feet did not reach the floor. The graph (Figure 3) makes these contrasts plainer than does the table.

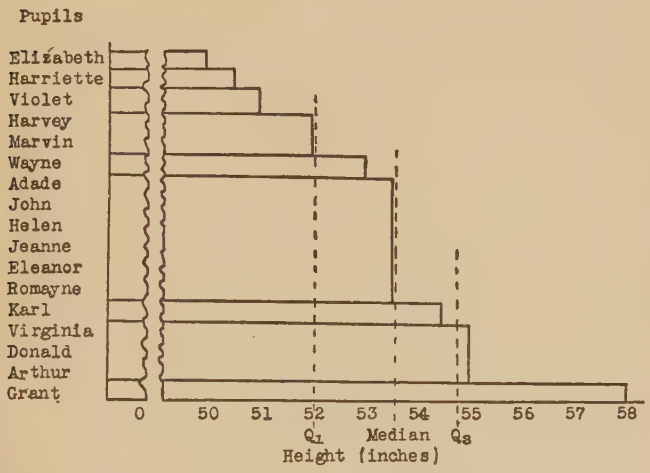

FIG. 3.—THE HEIGHT OF A SMALL FOURTH-GRADE CLASS AS MEASURED BY THEMSELVES IN ONE OF THEIR PERIODIC CHECKINGS OF PHYSICAL DEVELOPMENT.

Height is, of course, but one item in physical development. But it is a rough index of development, and under the suggestion of skillful supervision ought to lead to more complete checkings. In the minds of the committee, it served to show that their third thesis (II, 3) was very probably true, that school children seated in the same room and grade differ in the state of physical maturity they have reached.

4. AGE-GRADE DIFFERENCES

Mr. White provided substitute teachers to relieve the committee from their teaching duties when time was needed for research. They decided to begin at the first-grade rooms and visit each grade group including the senior class of the high school. The first impression received was that the differences in physical development, which they had noted in the fourth-grade room as discrepancies in height, were found everywhere. They were most notable, perhaps, in the middle and upper grades; and the superintendent's attention was called to two six-foot gentlemen in the seventh grade who, the committee believed, should be escorted to the high school and given an opportunity to live their school life with their physical equals.

The committee began to wonder whether the more subtle differences in mentality and educational achievement were not to be found also, as well as obvious physical differences. In fact, they wanted to know whether their fourth thesis (II, 4) was true.

School children classified in the same grade differ in age, and those of the same age are doing work in different grades.

They went back to the seventh-grade room and asked how old one of the six-footers was. The seventh-grade teacher, after looking on her register, replied that he was seventeen. Then they asked her to find the ages of all the other pupils, and they

arranged them on the blackboard as in Table IV and Figure 4. The teacher, as well as the committtee, was astounded to find that there was a difference of seven years in life age in her group of 47. It was hard to believe, too, that (using Ayres' standard of

TABLE IV

THE LIFE AGES OF 47 PUPILS IN A SEVENTH-GRADE ROOM IN THE MASON CITY SCHOOLS (NORM, TWELVE TO THIRTEEN YEARS INCLUSIVE).

Life Age (Years)	Frequency	Per Cent
1	2	3
17	1	2.1
16	3	6.3
15	2	4.3
14	9	19.2
13	9	19.2
12	9	19.2
11	11	23.4
10	3	6.3
Total	47	100.0
Retarded	15	31.9
Normal	18	38.3
Accelerated	14	29.8
Total Range	7	

two years' leeway to the grade) there were but 38 per cent who were just the right age (normal) for their grade and less than a third who were accelerated.

The committee decided to check this relationship of age to grade advancement throughout the schools. They could see that, if all pupils entered grade one at six years of age and progressed through one

grade each year without exception, all would be of the same age in every classroom which seated pupils of but one grade. But this certainly was not the case in the seventh-grade room they had examined. In

Fig. 4.—The Life Ages of 47 Pupils in a Seventh-Grade Room in the Mason City Schools (Norm, Twelve to Thirteen Years Inclusive).

brief, after receiving reports from all the teachers and spreading all the figures on a two-way chart according to the life age and grade placement of each pupil, they found only 65 per cent of the 1,200 pupils in just the right grade for their age, 23 per cent older than they should be (retarded), and but 12 per cent younger (accelerated).

In fact, it turned out that the committee had found confirmation for the fourth hypothesis (II, 4) which they had set down at the beginning of their work, and that it appeared that school children classified in the same grade differed in life age and those of the same age were located at many different grades of advancement.

5. SCHOOL ACHIEVEMENT

The next tentative generalization the committee examined had to do with differences in school success among the pupils of a single grade (II, 5).

School children in the same grade differ in their reaction to school subjects.

One member of the committee was a first-grade teacher, and she said that she believed they would find pupils reacting on different levels of success throughout the system about as she had discovered in her group of 40. She and her principal had taken for their own indoor "recreation" during the first month the task of counting all of the words in the ten primers and first-grade readers in the required list. Their work had resulted in weighted lists of the vocabularies of the books. This showed, for each book, each word in the book preceded by a figure giving the number of times it was found (frequency) in the stories in the book. After the class had finished reading a book, words of different frequencies

(weights) were used in simple stories, ten for each story. These stories were composed to represent the style of the stories in the book as nearly as possible and were printed in physical form identical with that

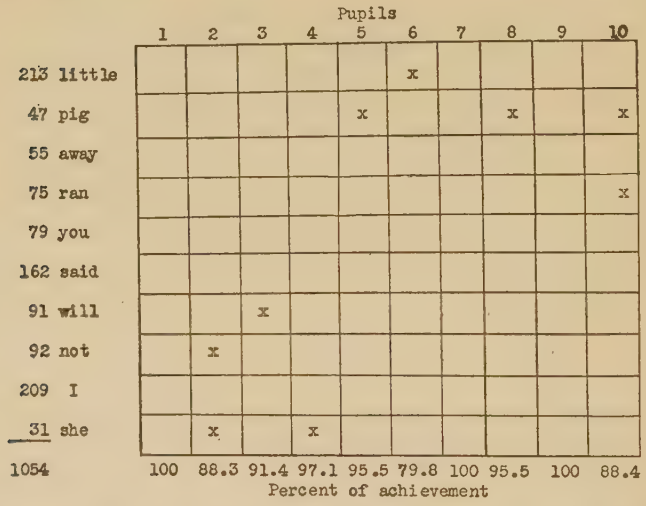

Fig. 5.—The Success in Word Recognition of Ten Pupils Selected at Random from a First-Grade Room of 40.

of the book (type, spacing, etc.). A chance sampling of ten children was taken in the room, and as each pupil read the story his success or failure in recognizing the ten selected words was noted on a check sheet like Figure 5.

It will be understood that missing a word like "little," which had been met 213 times in reading the primer, is a more serious error than failing to

recognize "will," which has a frequency of but 91. So, although child number three and child number six each missed but one word, the achievement in the one case may be represented by 91.4 and in the other by 79.8. It is easily seen that there is a wide range of difference here, even in this small group of ten children, a difference represented by the numbers 79.8 and 100.

The committee persuaded Mr. White to get supplies of standard tests in reading and to use them in all grades from one to twelve inclusive. When the results were finally in, they showed that the differences in reading achievement found in grade one were repeated in all reading classes, and later use of the Stanford Achievement Test showed similar differences when other school subjects were tested. In fact, thesis number five (II, 5) came out of this testing program as representative of the truth of the situation. It appeared that school children do differ in their reaction to the school curriculum, even children in the same grade.

6. LIFE AGE AND INTELLIGENCE

The work of the committee went along very well during the first semester. Quite a large number of the teachers, in particular in the central building, became deeply interested in the data which were handed to the superintendent from time to time and which he mimeographed for general distribu-

tion to the entire corps. Miss Brown, the seventh-grade teacher whose pupils had been shown to occupy such a bad eminence with regard to retardation (Table IV and Figure 4), called the upper-grade teachers into her room one day after school and asked them to check up age-grade facts in their rooms also, promising to get from the high-school principal figures for the ninth grade and data on elimination at the eight-to-ninth grade gap. They found a large percentage of the pupils too old for the grade and a mortality between the elementary and the secondary school of about 25 per cent.

Miss Brown had a talk with Mr. White about this. He, of course, knew about the junior high-school movement and intended to reorganize on that basis (6-3-3) at the proper time. He asked Miss Brown to serve on the survey committee and proposed to the committee that they make a study of the literature on the junior high school and of local conditions in grades seven, eight, and nine during the second semester. He told them, also, that some time after Easter he wanted to provide substitutes so that they could visit the junior high school in the neighboring system of Big Lake.

After the committee had read a good deal about the junior high-school age and junior high-school organizations, they asked the superintendent whether they might have a small budget to buy testing material. Further, they inquired whether his secretary would have time to help score the tests and tabulate

the results. What they proposed to do was to make a rather intensive checking of the capabilities and capacities of the 112 pupils enrolled in grades seven, eight, and nine (the logical junior high-school grades) in the central building. They decided first of all to make a preliminary study of a random sampling of the entire group, and for this purpose selected every other name from the alphabetical rolls in the teachers' registers. They bought a group intelligence test for this study and repeated the achievement test which had been given in the fall, using a second form. In addition to native intelligence and educational achievement, three other variables were measured—chronological age, home background, and physical development. To check the social and economic levels of home life, Dr. Heilman's revision of the Chapman socio-economic scale [1] was used, and the factor of physique was measured in terms of the ratio between height and weight.

All of the results of this testing program cannot be reported here, but individual data for three of the variables (columns 2, 3, and 5) together with derivative measures (columns 4 and 6) are given in Table V. The principal inquiry the committee had in their minds was as to whether the proposed junior high-school group was to any extent homogeneous. They wanted to know in particular about individual

[1] J. D. Heilman, "Revision of the Chapman-Sims Socio-Economic Scale," *Journal of Educational Research,* Vol. 15, September, 1928, pp. 117-126.

TABLE V

AGE AND SCHOOL ACHIEVEMENT DATA FOR A CHANCE HALF OF THE SEVENTH-, EIGHTH-, AND NINTH-GRADE PUPILS IN THE CENTRAL BUILDING OF THE MASON CITY SCHOOLS[1]

Pupils	Life Age (months)	Mental Age (months)	Intelligence Quotient (3 ÷ 2)	Educational Age (months)	Achievement Ratio (5 ÷ 3)
1	2	3	4	5	6
NINTH-GRADE BOYS					
1	168	189	112.5	184	97.4
3	188	135	71.8	148	109.6
5	169	205	121.3	190	92.7
7	180	217	120.6	205	94.5
9	180	167	92.8	167	100.0
11	150	154	102.7	160	103.9
13	153	206	134.6	202	98.1
15	155	189	121.9	178	94.2
17	148	206	139.2	210	101.9
NINTH-GRADE GIRLS					
1	160	156	97.5	210	134.6
3	187	148	97.9	155	104.7
5	181	180	99.4	180	100.0
7	178	156	87.6	154	98.7
9	173	192	110.9	190	98.9
EIGHTH-GRADE BOYS					
1	194	147	75.8	150	102.0
3	175	166	94.9	159	95.8
5	144	196	136.1	204	104.1
7	163	161	98.8	165	102.5
9	165	157	95.2	166	105.7
11	177	167	94.4	171	102.4
13	151	178	117.9	181	101.7
14	151	194	128.5	171	88.1
15	160	160	100.0	163	101.9
17	165	203	123.0	196	96.6
EIGHTH-GRADE GIRLS					
1	160	147	91.9	156	106.1
3	162	164	101.2	167	101.8
5	170	181	106.5	174	96.1
7	167	168	100.6	183	108.9
9	171	186	108.8	178	95.7
11	172	162	94.2	165	101.9
13	178	194	108.9	187	96.4
15	166	159	95.8	175	110.1

TABLE V (*Continued*)

Pupils	Life Age (months)	Mental Age (months)	Intelligence Quotient (3 ÷ 2)	Educational Age (months)	Achievement Ratio (5 ÷ 3)
1	2	3	4	5	6
SEVENTH-GRADE BOYS					
1	156	161	103.2	159	98.7
3	160	160	100.0	165	103.1
5	166	190	114.5	175	92.1
7	168	165	99.8	175	106.1
9	140	151	107.9	163	107.9
11	146	159	108.9	175	110.1
13	145	150	103.4	151	100.7
15	132	168	127.3	184	109.5
SEVENTH-GRADE GIRLS					
1	153	161	105.2	190	118.0
3	147	204	138.8	172	84.3
5	144	178	123.6	186	104.5
7	132	160	121.2	174	108.8
9	192	158	82.3	160	101.3
11	131	182	138.9	187	102.7
13	154	141	91.6	186	131.9
15	140	161	115.0	174	108.1
17	143	184	128.7	192	104.3
19	132	173	131.1	169	97.7
21	153	155	101.3	167	107.7
23	129	182	141.1	192	105.5
25	171	135	78.9	133	99.9
27	151	175	115.9	192	109.7
29	161	162	100.6	170	104.9
31	145	159	109.7	170	106.9
Median	162.1	165.8	105.7	174.4	102.5
Mode	(2)	162.5	102.5	(2)	102.5
Standard Deviation (Sigma)	16.1	19.0	16.5	16.4	8.3
Average Deviation (A. D.)	13.4	16.1	13.5	13.1	5.4
Coefficient of Variation (V)	10.0	11.1	15.2	9.3	8.1

[1] The five arrays are grouped in class intervals of five units for the above computations.

[2] These distributions have more than one mode.

differences, variability, and variations in the functions measured. They thought, too, that the facts of relationships (correlation) among the factors checked might tell something about the feasibility of reorganizing as a junior high-school unit, in particular if they should employ the technic of multiple and partial correlation, which uses coefficients of higher orders to hold constant disturbing variables. They wanted to make a regression equation also with achievement ratio as the criterion for prognosis, so that they might make individual predictions of school success for each of the 56 pupils in Table V. Finally, they wanted to examine the type of distribution of each array to discover conditions of normality or of skewness. These points of inquiry would include, of course, the sixth hypothesis (II, 6) with which they had started their research work.

School children of similar life age differ in their mental development.

With the help of Mr. White's secretary, who had had the courses in statistical method at the university, the committee did all this. The results were intensely interesting to them, and they decided to report the data of Table V and Figure 6 together with the simpler facts of correlation between intelligence and school achievement to the superintendent for discussion at a general teachers' meeting.

At the meeting, mimeographed copies of the material collected were distributed, and the chairman of

the committee called attention to the summation figures at the bottom of Table V. She pointed out, in particular, that for the group as a whole there were no very striking discrepancies between central ten-

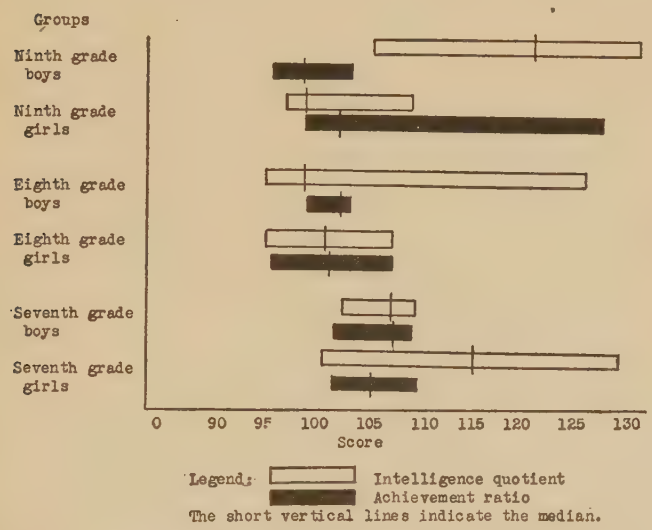

FIG. 6.—INTELLIGENCE AND ACHIEVEMENT LEVELS IN A GROUP OF 56 SEVENTH-, EIGHTH-, AND NINTH-GRADE PUPILS IN THE CENTRAL BUILDING OF THE MASON CITY SCHOOLS IN TERMS OF THE MIDDLE 50 PER CENT OF THE DISTRIBUTION OF THE TWO FACTORS FOR BOYS AND GIRLS IN EACH GRADE.

dencies (median and mode) when chronological age (column 2) and intellectual development (column 3) were compared and that a similar comment could be made when capacity (column 3) and actual achievement (column 5) were compared. The quotients and

ratios were very close to normality (100) in both cases.

Further, Figure 6 seemed to show, on the whole, a tendency toward horizontal homogeneity in the grades constituting the group when achievement was considered separately, although wider spreads in mentality appeared. And the backwardness in achievement shown for eight- and ninth-grade boys was very probably a criticism of the dry-as-dust curriculum offerings of the school, not of the pupils themselves.

Columns 2 and 3 were examined closely, and confirmation was found for hypothesis number 6 (II, 6). Many pupils of the same life age varied widely in mentality.

When the relationship of mental age to educational age was computed, the coefficient of correlation of order zero (r_{35}) was found to be $.707 \pm .045$; and the true relationship, in terms of the coefficient of partial correlation of the first order, when life age was held constant ($r_{35 \cdot 2}$) was $+ .709$. It appeared also that the socio-economic level of the homes of these pupils, as measured with Heilman's revision of Chapman's scale, did not have much effect upon their school work and that there was a significant relationship between intelligence quotient and achievement ratio.

The chairman explained also that the figures for spread (*sigma* and *A.D.*) and for variation (*V*) seemed to show possibilities of intellectual develop-

ment and achievement in the group, as the derivative measures (columns 4 and 6) were smaller than original data. Further, the very fact that the surfaces of frequency of the curves were far from normal very possibly pointed to a diversity of interest and capability characteristic of early adolescence. (Skewness: column 4, + .4909; column 6, + .2169).

All of the arguments pro and con cannot be given here, but the final consensus at the teachers' meeting was that further study should be made by the committee with a view to the possible organization of a junior high school (6-3-3 plan) at the beginning of the next school year. In particular, it was recommended that such factors as relative pupil-unit costs, housing possibilities, and availability of faculty should be carefully studied. And Mr. White pointed out that the distinguishing feature of the new unit, if it should come, must be a carefully differentiated curriculum selected in the light of definite knowledge about social objectives, and organized psychologically after a careful personnel study so as to fit the capacities, capabilities, and interests of the learners.

7. DIFFERENCES IN THE INDIVIDUAL

It will not be possible to follow the details of the studies made by the survey committee in Mason. The longer they worked the more convinced they became that, while group data were highly significant in revealing problems which must be studied in

the school system, actual solutions must be in terms of the individual child. Their interest, as usual during the beginnings of an objective attack on school difficulties, was first of all a matter of sympathy for the "under dog," and they began by making lists of all pupils retarded two years or more in every grade. It turned out that the majority of these were in the middle and upper grades of the elementary school, and a recommendation was sent to Mr. White that he make some arrangement for giving these backward pupils individual attention. During the second semester, twenty of these retards from grades five and six were put in a small room in charge of a skillful substitute teacher who was employed for full time. One important interest the committee had in these children was expressed in their seventh hypothesis (III, 7).

Chronological age and mental age differ in many school children.

The record of about half of this group, taken alternately from the class register list, is shown in Table VI. They were chosen for membership in the Coach Room on the basis of disparity between life age and grade placement. The table shows that this is accompanied by a marked discrepancy between age and mentality as well. The latter is uniformly lower, and their ratio (IQ) goes as low as 51 and no higher than 86 with a median of but 77.

The committee recognized that here was a striking illustration of their seventh tentative thesis (III, 7). In the case of these pupils at least, there were marked differences between individual chronological

TABLE VI

THE RELATIONSHIP BETWEEN CHRONOLOGICAL AGE AND MENTAL AGE IN THE CASE OF ELEVEN "BACKWARD" PUPILS ENROLLED IN THE COACH ROOM OF THE MASON CITY SCHOOLS

RETARDED PUPILS	LIFE AGE		MENTAL AGE		INTELLIGENCE	
	Rank	Years and Months	Rank	Years and Months	Rank	IQ
1	2	3	4	5	6	7
A	1	16–3	11	8–4	11	51.2
C	2	15–2	3	11–6	7	75.8
E	3	14–2	1	12–3	1	86.4
G	4	14–0	9	8–11	10	63.6
I	5.5	13–7	2	11–7	2	85.2
K	5.5	13–7	8	9–2	9	67.5
M	7	13–2	6	9–8	8	73.5
O	8.5	12–2	6	9–8	4.5	79.4
Q	8.5	12–2	6	9–8	4.5	79.4
S	10	11–8	4	9–11	3	85.0
U	11	11–1	10	8–8	6	77.3
Approximate Median		13–7		9–8		77.3
Total Range		5–2		3–11		35.2

and mental age; and a further expression of this difference was found in the low correlation of the two, which was found to be but .09. (Rho transmuted to r for columns 3 and 5 is $r_{35} = .0955$.)

8. INDIVIDUAL ACHIEVEMENT COMPLEXITY

In talking with Miss Green one day, Mr. White said: "Very probably you realize that perhaps the most significant thing your committee has done for the improvement of teaching in Mason is to call the attention of the corps to the differing abilities found in the personality of every school child in the system. We must teach, of course, not in terms of the group nor of the individual pupil in the group, but in terms of each fact of every child's complex achievement possibility. Your committee understood this when you recommended that I organize our Coach Room, but I want every teacher in the system to understand and have the 'coach' attitude and technic. I wish you would prepare further data illustrative of the complexity of pupil psychology and the ragged advancement which a pupil makes in his achievement responses to our curriculum. You might take your illustrative material from the Coach Room groups. Very probably you will find there the extremes of individual differences within the child himself."

Miss Green realized that the superintendent had in mind the eighth tentative thesis (III, 8) with which the committee had started their work.

Abilities and groups of abilities differ in many school children.

As pupils were shifted into the Coach Room for individual diagnosis and remedial teaching, it was

very easy for Miss Green and her committee to make frequent report of facts of complex personality such as the superintendent desired. But two of these

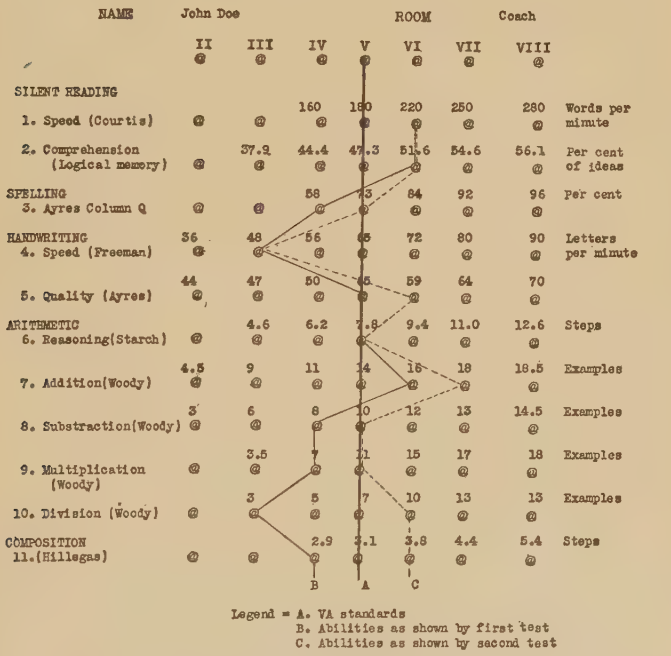
NAME John Doe
ROOM Coach
II III IV V VI VII VIII
SILENT READING
1. Speed (Courtis) 160 180 220 250 280 Words per minute
2. Comprehension (Logical memory) 37.2 44.4 47.3 51.6 54.6 56.1 Per cent of ideas
SPELLING
3. Ayres Column Q 58 73 84 92 96 Per cent
HANDWRITING 36 48 55 65 72 80 90 Letters
4. Speed (Freeman) per minute
44 47 50 55 59 64 70
5. Quality (Ayres)
ARITHMETIC 4.6 6.2 7.8 9.4 11.0 12.6 Steps
6. Reasoning(Starch)
4.5 9 11 14 16 18 18.5 Examples
7. Addition(Woody)
3 6 8 10 12 13 14.5 Examples
8. Substraction(Woody)
3.5 7 11 15 17 18 Examples
9. Multiplication (Woody)
3 5 7 10 13 13 Examples
10. Division (Woody)
COMPOSITION 2.9 3.1 3.8 4.4 5.4 Steps
11.(Hillegas)
B A C
Legend = A. VA standards
B. Abilities as shown by first test
C. Abilities as shown by second test

FIG. 7.—PUPIL ACHIEVEMENT RECORD OF JOHN DOE.

reports can be mentioned here. Figures 7 and 8 are typical. Figure 7 gives the details of the advancement made by one of the Coach Room boys during the six weeks between the initial difficulty analysis of his training status (line B) and his final testing

(line C) as he was leaving the group. Line A is drawn in accordance with the achievement norms

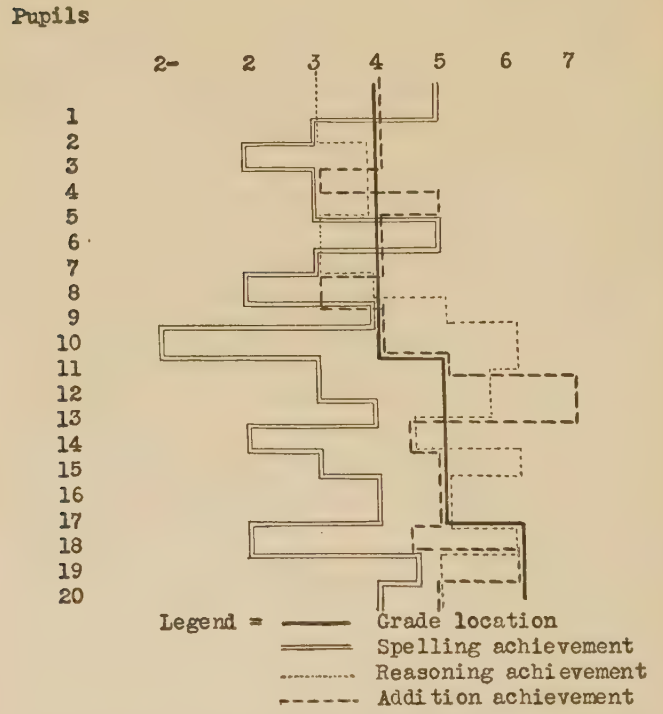

Fig. 8.—Grade Location, Ayres Spelling Achievement, Starch Reasoning Achievement, and Woody Addition Achievement of Twenty Pupils Retarded Two Years or More.

of thousands of other fifth-grade pupils in the eleven standard tests used. It will be seen that the profiles of achievement revealed by both the diagnostic and

the final tests are very uneven. In reading ability, John was found to be above the average (items 1 and 2), and very probably this explains in part his rapid advancement in functions 3, 5, 7, 8, 9, 10, and 11 where his teacher helped him to put the emphasis of effort. Evidently she thought energy should be expended at these points instead of in the handwriting segment of John's personality. At least she failed to convert him from his habitual slow drawing type of writing.

The entire Coach Room group, as found there during the second half-semester period of its organization, is included in Figure 8 in terms of levels of achievement in three important school subjects. The width of the graph indicates the spread of achievement in the case of each pupil and for the class as a whole. It is significant to find that, when but three facets of total personality are considered, pupils numbered 18 and 10, for example, exhibit a range of complexity reaching over four and more than four grades, while the achievement spread of the entire class is from less than the second-grade level to seventh grade, a total range of over five grades.

In fact, one finds here good evidence to substantiate the last tentative generalization with which Miss Green's committee began their work (III, 8). A measurement of specific abilities reveals significant differences even in the personality of the individual school child himself.

It will be impossible to follow the details of pro-

fessional development in the Mason school system farther. It was typical of that found in hundreds of small mid-western communities. The brief account given here ought to furnish sufficient information to give a very good conception of procedure in similar school situations in many states.

9. WHAT TO DO

As you re-read this first chapter, you should make a list in your notebook or on page 38 of all of the words you do not understand. Look up all but the statistical terms in a good dictionary. All of the terminology referring to educational statistics (some 62 words) is in the glossary at the end of this book. It will not pay you to find these in the glossary because, even though the definitions there and the nine formulas immediately following were written as carefully as possible, they will not enable you to understand the terms fully. This is the purpose of the chapters that follow.

This purpose is not to tell you anything about the derivation of the formulas discussed or even to give you any skill in the use of the 62 statistical words scattered through the exposition. It is believed, though, that a reading knowledge will be gained, if you follow this textbook carefully and do everything suggested. This reading knowledge will make it possible for you to meet the requirements of the education and educational-psychology courses, which

you will carry through the four or more years of your teachers-college experience, on a higher level of understanding. It will help you also to know what your instructors are talking about and to take part in class discussions.

This first chapter, then, serves as a diagnostic difficulty analysis. It tells you which of the statistical terms you don't know. Eleven of these will be used in the next chapter, and the rest of the fifty-one will be scattered through Chapters III to VII. Be looking for them as you read. If you don't understand them from their context and the explanations given, ask your instructor. Don't leave one unconquered.

I once knew a promising youngster about twelve years old who, when he was reading silently, would say when he came to a word which he could not pronounce and did not know, "Adam's off ox." His sister would hum three m's in a like emergency. Don't transmute this book into an agricultural treatise or a vocal exercise by such mental laziness!

Words in Chapter I Which I Do Not Understand

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50. .....

CHAPTER II

GETTING THE FACTS FOR A STATISTICAL REPORT

When you begin to teach, it will often be necessary to have definite information about your environment expressed in terms of amount, or figures. Number is, of course, always an element in any situation. You and your training teacher, when you begin student teaching, will be two. There may be forty desks in the classroom and thirty pupils, twelve of whom are boys.

1. THE TEACHER AS STATISTICIAN

Now, there are many types of facts with which a teacher deals in the everyday business of the classroom. Even in departmental work, she usually has some responsibility for a particular group. It is often valuable to make simple *studies* of differences among groups; for example, to find out how one fifth-grade class compares with another fifth-grade class across the hall in certain *traits* or *functions*, such as ability to work a particular list of examples in fractions or to read silently a short story found in a current magazine.

It may be that you will wish to find out something definite about each of the thirty pupils in your group, for you will learn while in the teachers college that the basic problem which the teacher must solve in the typical American classroom is how to handle children in groups and to teach them in terms of the individual pupil. You may decide to determine the *frequency* of such a *factor* as the successful spelling of a list of 100 fifth-grade words which occur in the English curriculum. Then you might make a *sampling* of the entire class of thirty in terms of the ten pupils who spelled the fewest words correctly, and using this *selected group* check up their speed in handwriting on the theory that their low achievement in spelling might have been caused in part by slow writing.

In fact, the typical activities of the competent teacher consists of series of *investigations* which give him diagnostic information about the situation in which he finds himself. If he is dull and incompetent, he is never aware of many of these problems, and never tries to solve them. To learn to recognize and to deal with the most important of the problematic situations of the schoolroom is one of the purposes of residence in a teacher-training institution.

The principals, the superintendents, and the teachers in school systems are, of course, interested in the same data about individual pupils and small classroom groups. They must also know something about total *populations* in the field of the

public school; for example, about all pupils ten years of age or about all high-school graduates. They must also check groups of data about total situations in order to get information of use in administering units of the organization in which they are working. For example, the superintendent of schools where you begin to teach suspects that not only expediency but the intrinsic worth of the plan make it imperative that he should change from the 8-4 scheme of organization to the 6-3-3 plan. In order to prove this to himself and to get information to take to the board of education in making his recommendation, he makes a study of typical junior high schools in systems of similar size to his.

2. METHODS OF RESEARCH

A superintendent may use three methods in an investigation such as this. He may: (*a*) read and abstract all studies in the junior high-school field which he finds in educational literature; he may (*b*) get information through the mail by sending inquiries, or *questionnaires*, to principals of junior high schools; or he may (*c*) get into personal touch with the schools by visiting them. The last method is, of course, the most businesslike and efficient; but usually it costs most. Ordinarily, in a case like this the student administrator looks into his bibliographies and begins reading and re-reading, starting at the same time a preliminary *questionary* study

among junior high schools and following this by as many visits to typical schools as funds will permit.

If you ever have to use the *question list* method of investigation, the rule is *Don't do it.* But this is, of course, nonsense; quite often there is no alternative between rather complete ignorance and some definite knowledge through the use of a question blank. What is meant is that the questionnaire is a very clumsy tool, very hard to arrange so that the actual facts desired will be forthcoming. In the first place, the word *questionnaire* is not highly regarded among a great many school men. If you must use this method, use some other word, "inquiry" or "question list," for example. In the second place, the returns from questionnaires are seldom over 50 per cent of all sent out, at least in the first call. And, finally, it is very difficult indeed to make a good question list in any field of inquiry.

The details of a good questionary cannot be gone into here, but you should be familiar with one of the most recent brief statements of the principles and technic of questionnaire construction. A veteran educator, Dr. Whipple, in a recent issue of *School and Society*,[1] says that a good questionary should conform at least to these stipulations:

1. It should be within the comprehension of those who are to answer it.

[1] G. M. Whipple, "The Improvement of Educational Research," *School and Society,* Vol. 26, August, 1927, pp. 249-259.

2. It should demand a minimal amount of writing.
3. It should be directed primarily to matters of ascertainable fact and less often to matters of opinion.
4. It should elicit unequivocal replies, especially if these are later to be subjected to statistical treatment.
5. It should deal with matters that are worth investigating and that will seem to the recipients to be worth investigating.
6. Though demanding only brief replies, it should stimulate supplementary communications from the recipients.
7. It should promise the respondent a copy of the published results.

3. ILLUSTRATIVE QUESTION LISTS

Illustrations of real question blanks recently used in the teachers-college and junior-college fields are reproduced as Tables VII and VIII. That on salaries yielded all of the information needed in making a complete report on salaries, salary schedules, and all other matters of payment to faculties in state teachers colleges.[1] That on the junior college covers but a part of the data included in the study.[2] All of the previous investigations in the field were carefully read for procedures and technics and to get

[1] E. Lowell Kelly, *Salaries in State Teacher Training Institutions,* Research Bulletin No. 13, April, 1928 (Department of Educational Research, Colorado State Teachers College, Greeley, Colorado).

[2] F. L. Whitney, *The Junior College in America,* Colorado Teachers College Education Series, No. 5 (Colorado State Teachers College, Greeley, Colorado, 1928).

TABLE VII

A Question List Used in a Recent Teachers-College Salary Investigation

1. Do you have a salary schedule? ..... Yes No
   If answer is "Yes," please report details in a letter or by sending printed mimeographed material.
2. Do you grant leaves of absence to faculty members? ..... Yes No
   (*a*) How often? .....
   (*b*) For how long? .....
   No pay ( )
   (*c*) Irregular? .....
   (*d*) Full pay ( ) Half pay ( )
3. Are salaries paid for time off during illness or for other causes? ..... Yes No
   (*a*) For how long? .....
   (*b*) Full pay ( ) Half pay ( )
4. Do you pay a bonus for attendance at summer school sessions? ..... Yes No
   Do you pay a bonus for attendance at educational association meetings? ..... Yes No
5. Are faculty members paid extra for extension work? (*a*) Teaching ..... Yes No
   (*b*) Correspondence ..... Yes No
6. What is the annual cash salary of the president? $.....
   Additions: House $..... Car $.....
   Traveling Expenses $..... Other items $.....
7. Fill in the table below including all the faculty except the president. Give: (*a*) the number of faculty members receiving each amount of salary for the school year 1927-1928, (*b*) the number of months in the salary year. If the faculty is not separated into the three grades indicated, include all except the training teachers in the column headed "Instructors."

TABLE VII (Continued)

Professors			Instructors			Training Teachers		
Salary $	(a) Number	(b) Months	Salary $	(a) Number	(b) Months	Salary $	(a) Number	(b) Months
4000-4099			3500-3599			3300-3399		
3900-3999			3400-3499			3200-3299		
3800-3899			3300-3399			3100-3199		
3700-3799			3200-3299			3000-3099		
3600-3699			3100-3199			2900-2999		
3500-3599			3000-3099			2800-2899		
3400-3499			2900-2999			2700-2799		
3300-3399			2800-2899			2600-2699		
3200-3299			2700-2799			2500-2599		
3100-3199			2600-2699			2400-2499		
3000-3099			2500-2599			2300-2399		
2900-2999			2400-2499			2200-2299		
2800-2899			2300-2399			2100-2199		
2700-2799			2200-2299			2000-2099		
2600-2699			2100-2199			1900-1999		
2500-2599			2000-2099			1800-1899		
2400-2499			1900-1999			1700-1799		
2300-2399			1800-1899			1600-1699		
2200-2299			1700-1799			1500-1599		
2100-2199			1600-1699			1400-1499		
2000-2099			1500-1599			1300-1399		
1900-1999			1400-1499			1200-1299		
1800-1899			1300-1399			1100-1199		
1700-1799			1200-1299			1000-1099		
1600-1699			1100-1199			900- 999		
1500-1599			1000-1099			800- 899		
1400-1499			900- 999			700- 799		
1300-1399			800- 899			600- 699		
1200-1299			700- 799			500- 599		
1100-1199			600- 699			400- 499		
1000-1099			500- 599			300- 399		
Total....								
Median ..								

(Signed .....

TABLE VIII

A Junior-College Question List Used in a Recent Study

Name of Junior College ..... Location .....
President, Principal, or Dean .....

Would you be willing to fill in the items above (if not complete) and to give the information asked for below? Please return this sheet in the stamped envelope enclosed. These data are needed in a nation-wide survey of the junior college which is being made for the state educational institutions of Colorado. Tabulated returns will be available as soon as all junior colleges have replied.

---

1. Kind of institution: Public .....; private .....; for men .....; for women .....; coeducational .....
2. Number of years of college work offered: One year .....; two years .....; two years only .....; a total of three years .....; a total of four years .....
3. Under what auspices or control is your college conducted? .....
   (For example: High school district, city school district, junior college district, state, private, Baptist church, Lutheran Synod, and so forth.)
4. What year was your junior college established? .....
5. When was your junior college accredited? ..... By what institution? .....
6. What credit do your graduates receive at your state university? ..... semester hours.
7. Number of semester hours required for graduation from your junior college .....; degree granted .....
8. Number of graduates: 1926-27 .....

9. Number of these graduates continuing college work in higher institutions .....
10. Enrollment: 1926-27, freshmen ..... sophomores .....; 1927-28, freshmen ..... sophomores .....
11. Please give the reasons for the organization of your junior college and its special purpose .....
12. Please send a copy of your latest catalog and course of study and any other printed or mimeographed material descriptive of your junior college.
13. Please make lists on the back of this sheet of all (*a*) the public and (*b*) private junior colleges in your state.

Very truly yours
FREDERICK L. WHITNEY, *Director*.
Department of Educational Research

comparative material to be included in the final report. The question list does not deal at all with the legal status of the junior college in the states where it is found. This had to be ascertained by briefing all junior-college laws found in the statute books. To the extent that funds would permit, the procedures of the investigation ought to have included visitation to typical institutions of the sort studied.

The first questionary conforms very well to the seven standards given by Whipple. Standards number 6 and 7 were taken care of in the letter of request and explanation which accompanied the list of questions. The junior-college question blank

comes nearer to being a complete inquiry in question form. Notice that every junior-college president or dean will be fully able to answer it (Standard 1), that very little actual writing is required (Standard 2), that no opinions are called for at all (Standard 3), that replies must be "unequivocal" (Standard 4), that the facts asked for are of vital moment to the junior-college authorities (Standard 5), that item 11 gives room for supplementary data (Standard 6), and that a copy of the results of the study is promised (Standard 7).

4. ILLUSTRATIVE EXERCISES

In order to get a clearer idea of the real meaning of a questionary, it will be a good plan for you and your instructor to work through the actual details of making one or two brief question lists.

a. Assume that the members of the class which is reading this book are all superintendents of schools in small cities of about 5,000 population and that your instructor is a superintendent also in a city of about the same size. He has been studying the junior high-school movement in the summer classes he attended at the state teachers college, and he wants information from you about conditions in your school systems. He thinks that this information will aid him in deciding what he should or should not do in his own case. He wants to know what type of organization of school units you have, whether

8-4, 6-3-3, 6-4-4, or what not. He wants to know why you have the organization you have; when you adopted it; why you have made no change, etc. He wants to know the details of your experience in reorganization, if you have adopted the junior high-school plan. This will include such items as how you sold the plan to your constituency, how you determined bases for reclassification, what survey data you secured before doing anything, etc., etc.

Make a brief question list which might be used as indicated above. Check it up with Dr. Whipple's seven criteria and be sure that none are violated.

b. Suppose the sociology class wants to make a study of some of the important factors in the home background of this year's freshman class. Arrange the following items in questionnaire form so as to conform to the seven principles given above: (*a*) nationality of father and mother, (*b*) education of father and mother, (*c*) father's occupation, (*d*) father's income, (*e*) mother's occupation, (*f*) mother's income, (*g*) number of children, (*h*) which child is the student?, (*i*) teaching experience of father and mother, (*j*) occupation and income of each child, (*k*) telephone, (*l*) automobile, (*m*) furnace, (*n*) daily papers and magazines taken (*o*) books owned, (*p*) radio, etc., etc. Do not put in your questionary any more than can be typed on one side of an 8½ x 11 sheet.

5. WHAT TO DO

Now check through this list of eleven terms that are often found in educational statistics. After reading the chapter, is some halo of meaning attached to each?

factor	selected group
frequency	study
function	questionnaire
investigation	questionary
population	question list
sampling	

If you have conquered them and they don't belong there, check them off the list you made on page 38.

But if any word means just nothing yet, go back and find it in the chapter. Find it also in Chapter I. Discuss it with your roommate. Ask your instructor. Be spurred by an irritating unrest until that word obediently dons its halo.

If an unknown statistical term appears in your required reading, look it up in the glossary of this book, read the definition, and turn back to the page indicated after the definition to see how it is used there. If this does not clear up its meaning, ask your instructor.

CHAPTER III

ORIGINAL ORGANIZATION AND CLASSIFICATION

It may be that in your reading of educational literature you will come across the twofold terminology which statistics sometimes uses to include all kinds of quantitative and qualitive measures. The two words are *attributes* and *variables*, and it will be best to understand them now.

1. ATTRIBUTES AND VARIABLES

An attribute is usually considered to be a quality, trait, or function which is present or absent. On the other hand, a variable simply exists in different amounts. That is, an attribute has to do with the grouping of the individuals studied according to the presence or absence of a particular quality or trait. Sometimes several traits are involved, and the presence of one or another of them is considered. But usually the method of attributes involves a twofold *classification* which divides into two groups or classes. For example, your report to the principal may classify your class as boys and girls, as 6A group and 6B group, as satisfactory or unsatisfac-

tory in school progress, etc. But, as has been intimated, it may include more items of classification: normal, slow, and fast progress; normal, over-age, and under-age in grade placement; and the like.

A variable has to do with the quantity of a trait, quality, or function found in any study. This quantity usually depends upon the conditions under which the trait is measured, time being one of the conditions. For example, the life age of your pupils varies as you report it term after term, and it varies also to correspond to the different time each was born. The marks you give John in his monthly arithmetic tests vary, and all of the marks you give the thirty pupils in each examination are as a whole different from those of any other examination. It may be that they are getting steadily better as the weeks go by.

2. ORGANIZATION AND CLASSIFICATION

Now, as you read the last two sentences in the paragraph above, you may have had a feeling of deficiency and indefiniteness in the statement made. If the groups of arithmetic marks are different and are "getting steadily better" in successive tests, how much better are they each time?

Suppose you should decide to give several tests in your classroom of thirty 6A and 6B pupils. You have them write an essay examination in geography first. Then you give an intelligence test, a standard test in fractions, and a new-type examination in civ-

ics which you have made yourself. When all of the papers have been scored and are piled on your desk in four piles, the first logical thing to do is to *organize* them in terms of the big differences present. You take the pile of intelligence tests, for example, and separate the booklets first into 6A and 6B piles, then each of these groups by sex.

Perhaps organization stops there, and classification begins. It is not important to try to decide just at what point this transition takes place. If you want to get the real facts of the situation, as revealed by the tests given, you simply keep on arranging and reorganizing the scores on succeeding levels of generalization.

Suppose that you used the Otis Classification Test of Mental Ability, and the 6A scores proved to be

16, 37, 21, 37, 46, 68, 54, 53, 21,
16, 54, 53, 52, 47, 37, 65, 58

Without some rearrangement of these scores and without reading all seventeen of them, you cannot speak of them all at once or say anything about them as a group. But you can describe the group in terms of the largest and the smallest scores and say that they *range* in value from 68 to 16 and that the total range is 52 units.

But while this is somewhat better than the original conglomerate, it is not much better and does not tell anything very definite about the intelligence of the 6A group as a whole. Having found the largest and

the smallest scores, you try putting them all down in order of value, thus:

68, 65, 58, 54, 54, 53, 53, 52, 47,
46, 37, 37, 37, 21, 21, 16, 16

This makes a more orderly arrangement, and when the *array* is tipped vertically it is easy to indicate the *rank* of each score.

Rank	Scores
1	2
1	68
2	65
3	58
4.5	54
4.5	54
6.5	53
6.5	53
8	52
9	47
10	46
12	37
12	37
12	37
14.5	21
14.5	21
16.5	16
16.5	16

This array has seventeen items, and if it were much larger it would be awkward to try to put it

on a long paper as a unit. It will be noticed that there are three scores of 37, and it might be possible to use but one line in the array in such cases, telling the number of scores in such groups and giving them the rank of the middle of the group thus:

Rank	Scores	Frequency
1	2	3
1	68	1
2	65	1
3	58	1
4.5	54	2
6.5	53	2
8	52	1
9	47	1
10	46	1
12	37	3
14.5	21	2
16.5	16	2
Total		17

But a better way is to organize the facts at once as a *table*, grouping the scores by *step intervals*, or *class intervals*, as in Table IX. The *distribution* here appears in terms of frequency of scores by class intervals. There are two scores between values 65 and 70, none from 60 up to 65, one from 55 to 59 inclusive, etc. The total frequency is seventeen scores, one for each pupil in the group.

Now, you have a picture of the mentality of this 6A-group in terms of the scores they made in the

Otis test of mental ability. But perhaps you have noticed that it is not possible even yet to say anything in general about the group, except that the scores have a total range of 52 units on the scale. This deficiency will be discussed in Chapter V.

TABLE IX

THE DISTRIBUTION OF MENTAL ABILITY IN A GROUP OF SEVENTEEN 6A PUPILS AS MEASURED IN TERMS OF THE AVERAGE SCORES IN FORMS A AND B OF THE OTIS CLASSIFICATION TEST

Class Intervals	Frequency
1	2
65–69	2
60–64	0
55–59	1
50–54	5
45–49	2
40–44	0
35–39	3
30–34	0
25–29	0
20–24	2
15–19	2
Total	17

3. ILLUSTRATIVE EXERCISES

a. I am doing student teaching in the training school and handling a small group of fifteen fourth-grade pupils. My training teacher has let me take

almost full charge of their arithmetic class work. She asked me yesterday to write her a brief report of their progress during the last three weeks. She suggested that I report the rank of each member of the class in arithmetic, and compare this with rank in intelligence. We have their scores in a standard intelligence test recorded on their cumulative record cards. I have given an hour test over the arithmetic work of the three weeks' period, and the results are as reported below. Instead of giving the pupils' names, each individual is assigned a code number. These are enclosed in parentheses.

ARITHMETIC ACHIEVEMENT

(1) 80, (2) 80, (3) 94, (4) 96, (5) 75, (6) 80, (7) 97, (8) 75, (9) 94, (10) 94, (11) 82, (12) 87, (13) 91, (14) 93, (15) 80

The intelligence scores, taken from the files, are as follows:

INTELLIGENCE SCORES

(1) 140, (2) 126, (3) 126, (4) 145, (5) 150, (6) 120, (7) 140, (8) 140, (9) 150, (10) 127, (11) 125, (12) 137, (13) 131, (14) 134, (15) 130

Arrange these two arrays in rank order and report the rank of each pupil in each array.

b. My seventh-grade class of twenty pupils is increasing in speed of silent reading every month. The last time they asked for the stop watch to

check the number of words they could read per minute, their scores were as follows:

190, 170, 175, 180, 189, 185, 182, 191, 193, 174,
176, 175, 175, 186, 185, 186, 186, 180, 188, 171

Make a table of these figures, showing the distribution of the measures of this variable arranged in proper class intervals and also showing the total range.

4. WHAT TO DO

Now, check over these eleven terms which have been used in the above discussion:

array	range
attribute	rank
classification	step interval
class interval	table
distribution	variable
organize	

Cross each word off the list you made on page 38, if you have really added it to your personal vocabulary. Does each mean something quite definite to you? If you come across the word *distribution* or the words *rank* or *range* in your required reading, will the sentences in which they occur be clear? Don't be satisfied, if any of these eleven mean nothing. Try to get the habit of being insatiably curious until each new word you find obediently appears with its required halo of meaning.

CHAPTER IV

GRAPHIC PRESENTATION

As you read the required portions of educational journals and books in connection with your teachers-college courses, you will notice that the facts which the authors are presenting are ordinarily printed in two forms, as tables and as *graphs*, or *diagrams*. Those who are reporting the results of research in education find that very often it aids the discussion and interpretation of the facts which are being presented to reproduce certain parts (or sometimes all) of the more generalized tables in graphic form as *charts*, or *figures*.

1. THE FIGURE

A graph, chart, figure, or diagram is a picture of the data of a table or of a part of them. For example, the intelligence scores of the seventeen pupils shown in Table IX (page 56) may be shown in graphic form as in Figure 9. Notice that the number of pupils is shown on the vertical line to the left and that the class intervals appear below the bottom line. The picture makes it very clear that two pupils

have scores in the 15 to 19 step interval, two in the 20 to 24 interval, none in the next two intervals, three in the 35 to 39 step, etc. The heavy lines are drawn across the top of each interval column; and when they are all connected a continuous surface of frequency, or *curve*, is the result. The value of this is that one gets at a glance a generalized notion of the distribution of these seventeen intelligence scores in the class.

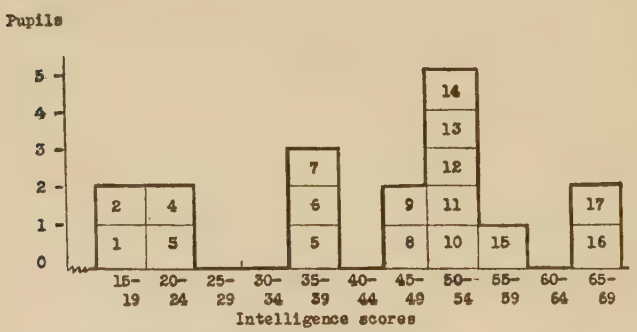


Fig. 9.—The Distribution of Mental Ability in a Group of Seventeen 6A Pupils as Measured in Terms of the Average Scores in Forms A and B of the Otis Classification Test.

In order to show just how a figure is made, suppose that the individuals in this same group of seventeen pupils are ranked in an array according to their scores in an arithmetic test, each score constituting one rank order. The facts of this ranking might appear as in Table X. The pupil who made the highest score is first in rank, score 94 is second, 84 with two pupils at that level is rank 5, etc.

TABLE X

SEVENTEEN 6A PUPILS DISTRIBUTED ACCORDING TO THE RELATIVE RANK OF THEIR ARITHMETIC SCORES

Rank	Arithmetic Scores	Number of Pupils (Frequency)
1	2	3
1	95	1
2	94	1
3	90	1
4	85	1
5	84	2
6	81	2
7	80	5
8	76	2
9	70	1
10	65	1
Total		17

Now, in making a picture of this situation, one starts from a point of origin, *O* (Figure 10), and draws a vertical axis, *OY*, and a horizontal axis, *OX*. Then the *OY* line is divided into five parts, as it is noticed that 5 is the largest frequency in column 3, Table X. And line *OX* is numbered at 10 points to represent the number of rank positions in column 1. To get the first point in the figure, go out on the *OX* axis to rank number 1 and then up to pupil frequency number 1. Put a cross at the intersection of the coördinate lines. To get the second point, count to the right two units from the point

of origin and up one. After this process is gone through for each of the ten pairs of facts in Table X, connect the crosses in Figure 10 with heavy lines, and the picture is complete. This is a line-graph.

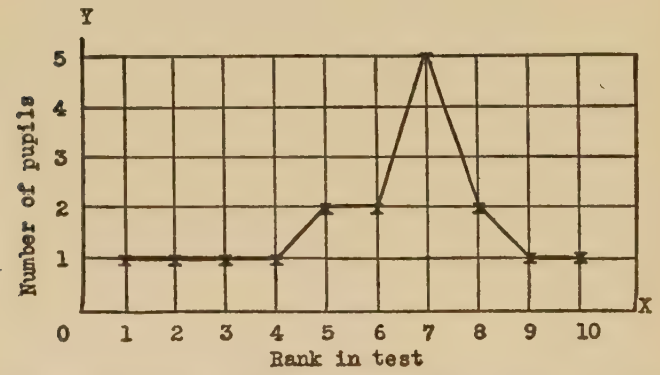


FIG. 10.—SEVENTEEN 6A PUPILS DISTRIBUTED ACCORDING TO THEIR RELATIVE RANK IN AN ARITHMETIC TEST.

There are other methods of drawing the surface of frequency in a figure. One may have a column graph, as in Figure 9, page 60, or the columns may be kept separate as in Figure 11, making a bar graph. Notice that the coördinate lines do not appear in Figure 11, nor in Figures 1 and 2 in Chapter I. This is the best method. All of the coördinate lines are not necessary, and they may interfere with a perfectly clear representation of the surface of frequency.

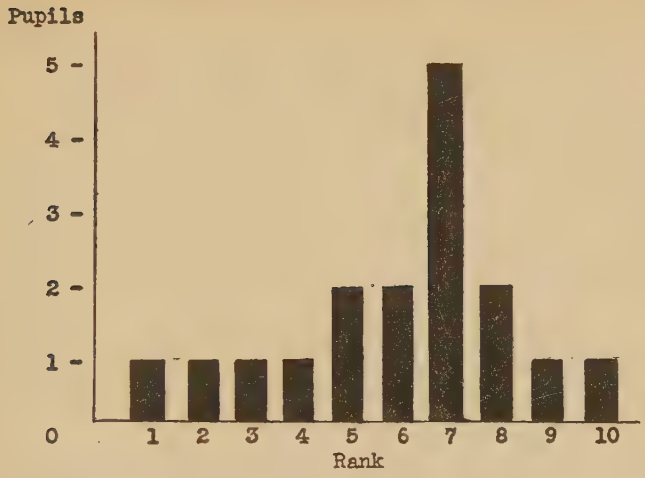


FIG. 11.—SEVENTEEN 6A PUPILS DISTRIBUTED ACCORDING TO THEIR RELATIVE RANK IN AN ARITHMETIC TEST.

2. NORMAL AND SKEWED CURVES

Physicists, biologists, and psychologists have found that, if they get a large number of facts about any situation they are studying, all of the measures will tend to group themselves around a large portion of them having about the same value. It would be clearer to say that they always find a few very small measures, more somewhat larger, a large number of just about average size, fewer somewhat larger than these, and a very few largest of all. This is true when we group measurements of original traits, or functions, in the inanimate, physical, mental, or spiritual realms. For example, the widths of

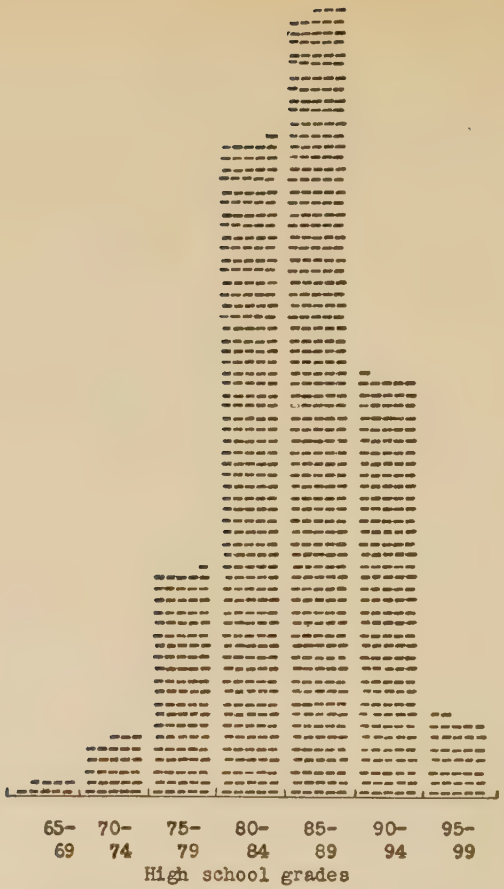


Fig. 12.—The Distribution of the Average of the High School Marks of 1,000 Graduates of Two Year State Normal School and State Teachers College Courses in Twelve States.

(F. L. Whitney, *The Intelligence, Preparation, and Teaching Skill of State Normal School Graduates in the United States.* Doctor's Dissertation, University of Minnesota, Public School Publishing Company, Bloomington, Illinois, 1922.)

1,000 of the leaves on a single tree have been found to distribute themselves in this way. The weight of men, memory for digits, and certain character traits do the same. Figure 12 shows how the high-school grades of 1,000 normal-school graduates are distributed. The frequency figures go from smaller to larger and from fewest, more, most, lesser, to less. Putting them in a row, by intervals, they look like this:

9, 28, 101, 291, 348, 186, 37

Notice that the figures of the first and the last intervals are not exactly the same. One is 9, and the other is 37. Likewise, the second interval has 28 cases, while the next to the last has 186. If these 1,000 measures were distributed symmetrically from smallest to largest, the figures for the seven intervals might look like this:

30, 110, 220, 280, 220, 110, 30

A distribution like this is called a *normal distribution*, or *normal curve*, because usually the related facts in any realm of investigation distribute themselves normally in this manner. The same sizes of interval groups, decreasing in number equally toward the smallest and the largest, are found on either side of a large central group. Putting these measures in a graph and drawing the base (OX) line and the surface of frequency only, we have Figure 13. If this chart be drawn as a smoothed

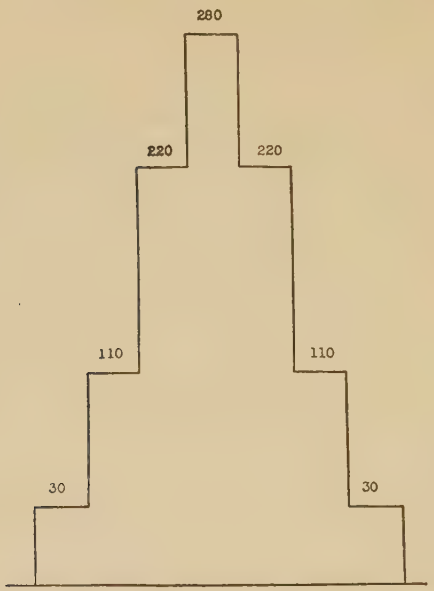


FIG. 13.—A COLUMN DIAGRAM SHOWING A NORMAL DISTRIBUTION OF 1,000 MEASURES.

line graph instead of a column diagram, it would look like Figure 14.

The fact that one finds data arranging themselves normally in this bell-like form is of great use to investigators in such social sciences as education. (You understand, though, that education is not yet a true science in the sense that physics is, for example. Many educators, however, are beginning to use the scientific method.) For one thing this fact makes it possible for investigators to assume that any group of facts with which they may be dealing

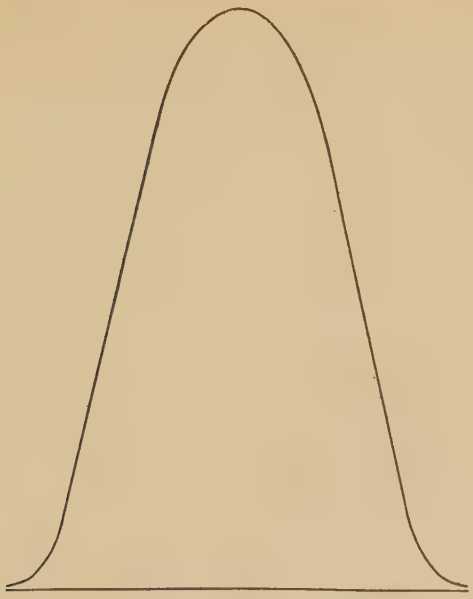


FIG. 14.—A NORMAL CURVE.

is normally distributed, provided they have included enough of them in their investigation. The sampling must be adequate, or normalcy cannot be assumed.

Not only research students, but supervisory officers and classroom teachers in the public schools as well, may profit from this peculiarity of large collections of data. For example, if you should be teaching geometry in high school and should save the term marks you give your classes for years until you have a very large number of them, you might expect them to distribute themselves under a normal

surface of frequency, if you arranged them in graphic form.

The fact is, though, that you very probably would not get a perfectly normal curve in a case like this, if the marks you have in geometry were of the ordinary sort, subjectively derived and influenced both by your character traits and those of your pupils. If you were a very conscientious teacher, there would tend to be more of the lower marks. If you were liberal in your attitude toward pupil effort, the marks might tend to pile up toward the better end of the distribution. That is, the distribution would be *skewed* one way or the other instead of being perfectly normal. In the case of overconscientiousness, the curve might look like Figure 15. The flat side is toward the right, and there is said to be positive skewness. In the case of extreme lib-

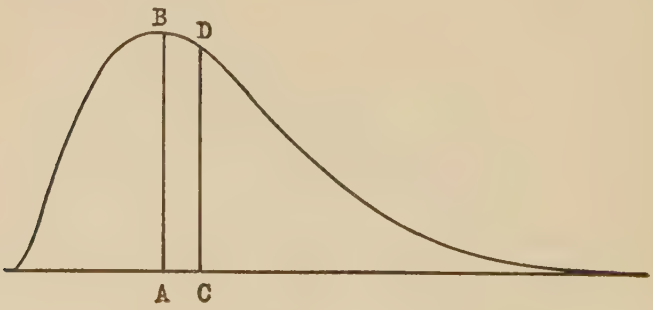


Fig. 15.—A Distribution Skewed to the Right (Positive Skewness).

erality, the surface of frequency might look like Figure 16. The flat side is toward the left, and this is said to be negative skewness.

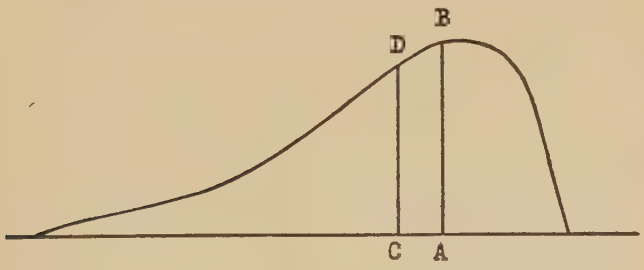


FIG. 16.—A DISTRIBUTION SKEWED TO THE LEFT (NEGATIVE SKEWNESS).

3. ILLUSTRATIVE EXERCISES

a. We had daily and weekly themes to write in two of my high-school English courses. Miss Smith taught both of these composition courses, and she was certainly not very liberal in the marks she put on our papers. I made a list of about two dozen of these grades.

60, 75, 60, 79, 80, 95, 90, 81, 85, 84, 73, 71, 72,
80, 73, 74, 61, 65, 63, 62, 60, 68, 67, 69, 67, 66

How do these look when represented as a column graph? Which kind of a "skewer (!)" was Miss Smith, positive or negative? Make a good title for the figure.

b. I had charge of the playground period for a group of thirty-six of the junior high-school boys yesterday. They were a lively bunch. I finally persuaded them to organize and compete for the running broad jump. Their distances were as follows:

17, 17.5, 16, 9, 10, 13, 12.5, 14, 14.5, 13, 13.5, 12, 12.5, 12, 10, 14, 10, 12, 12.2, 12, 14, 14, 14, 10, 10.5, 13, 13, 14, 12, 12.5, 13, 13.5, 12, 12.5, 13, 12

One of the boys made a bar graph of these figures, and brought it out to us to-day. What did it look like? Was the distribution almost normal, or was it skewed?

4. WHAT TO DO

Now read these seven terms which have been used in Chapter IV:

chart	normal curve
diagram	normal distribution
figure	skewness
graph	

Will each of the seven be clear enough, when you find it in your required reading, so that the meaning of the context will not be confused? If so, check it off the list you made on page 38.

It will be interesting, when you take your course in educational statistics later in your college experience, to learn more about these words and the facts which they represent. For the present, it will be enough if you have a speaking acquaintance with each rather than an intimate knowledge.

CHAPTER V

DETERMINING AVERAGES

Perhaps you have noticed that in Chapters II, III, and IV the groups of facts presented in tabular and graphic form were dealt with as units of the whole. When the descriptive data about any situation were spoken of, nothing analytical could be said. It was necessary just to speak of all of them together. This did not tell us anything very definite about the composition of any group of material.

1. THE MEAN

If I have a small number of measures like

25, 82, 76, 76, 34, 43, 55, 76, 66, 29, 27,

I can talk about them by just saying all eleven of them every time I refer to them. But this would be very awkward indeed. You know what any sensible person would ask about them. It would be, "What is the average?" Now, when one is speaking statistically, *average* is a term which means any expression of *central tendency* for a group of data. When we were in the sixth grade, we used to get the "aver-

age" by dividing the sum of a column of figures by their number. In educational statistics, this is called the *arithmetic mean*, or just the *mean*. For the small group of eleven measures above, it is

$$589 \div 11 = 53.55$$

2. THE MODE

But by just looking at the group of eleven measures, the central-tendency fact which appears right on the face of it and without any calculation at all is that there is a small group of three figures of the same value (76). This is called the *mode*. It is the most fashionable number. If these eleven measures were human beings, the other eight would feel uncomfortable until they would be as nearly like the 76's as possible! Have you ever read *Keeping Up with Lizzie* by Irving Bacheller? Sometimes a distribution has two modes (*bi-modal*), and in some curves there are many (*multi-modal*). For example, in this group of figures below, there are two fashion groups, the 99's and the 70's. It is bi-modal.

99, 99, 99, 99, 81, 80, 73, 71, 70, 70, 70, 65, 60

Then, there is a third measure of central tendency, or average, which you will find very often in modern educational literature. This is the *median*. (Notice, this is not medium!) If the eleven scores above are

arranged in order of value from largest to smallest like this:

82, 76, 76, 76, 66, *55*, 43, 34, 29, 27, 25,

the middle figure, or median, is 55. Notice that the number 55 is sixth in rank from either end of the array.

Now, suppose you wanted to tell your principal something about the achievement of your seventeen 6A pupils who took the arithmetic test as reported in Table X (page 61). He might say, "How did they do in that test?" "Oh, they did very well. John was marked 95," you reply. But you both realize that this is not a very illuminating remark; and you turn to the board, write the distribution of scores, and find the central tendencies as in Table XI. The arithmetic mean, sometimes called *arithmetic average*, turns out to be 81.24. The most fashionable score, *la mode*, is clearly one of the group of five 80's. The median is, of course, the middle score; and this is 80 also because, counting the frequencies of column 2 either up or down, the ninth falls in the 80 group.

This finding the three facts of central tendency, the three averages, is not quite so simple when the frequency of measures is reported by class intervals. Notice Table XII, which contains the same facts as Table IX (page 56). Here, the mean score in the mental test turns out to be 43.68. This is obtained by letting the midpoint of the measures in

each step interval (67.5, 62.5, etc.) represent the whole interval, and then multiplying by the frequencies before adding. The mode can be said to be some score in the 50-54 interval, if no information

TABLE XI

THE ACHIEVEMENT OF SEVENTEEN 6A PUBLIC-SCHOOL CHILDREN IN AN ARITHMETIC TEST

Arithmetic Scores	Frequency
1	2
95	1
94	1
90	1
85	1
84	2
81	2
80	5
76	2
70	1
65	1
Total	17
Arithmetic mean	81.24
Mode	80
Median	80

other than that in this table is available about the value of each individual score. Or the mid-point of the 50-54 interval (52.5) might be taken as the best single measure by which to represent the mode.

TABLE XII

THE DISTRIBUTION OF MENTAL ABILITY IN A GROUP OF SEVENTEEN 6A PUPILS AS MEASURED IN TERMS OF THE AVERAGE SCORES IN FORMS A AND B OF THE OTIS CLASSIFICATION TEST

Class Intervals	Midpoints	Frequency
1	2	3
65–69	67.5	2
60–64	62.5	0
55–59	57.5	1
50–54	52.5	5
45–49	47.5	2
40–44	42.5	0
35–39	37.5	3
30–34	32.5	0
25–29	27.5	0
20–24	22.5	2
15–19	17.5	2
Total		17
Mean		43.68
Mode		50–54 (52.5)
Median		48.75

3. THE MEDIAN

The median, when data are reported as in Table XII, is obtained by assuming that the measures in the middle class interval are uniformly distributed through that class and finding the midscore of the entire distribution.

It is not the purpose here to learn how to do this, but the problem is worked out in case you have a curiosity to know the details.

$$\text{Median} = \text{Lower limit of middle class} + \frac{\frac{N}{2} - \text{Partial sum}}{\text{Class frequency}} \times \text{Class interval}$$

$$\text{Med.} = 45 + \frac{\frac{17}{2} - 7}{2} \times 5 =$$

$$45 + \frac{1.5}{2} \times 5 = 45 + 3.75 = 48.75$$

TABLE XIII

THE AVERAGE SPELLING ACHIEVEMENT (PER CENT OF WORDS CORRECTLY SPELLED) AMONG 96 ELEMENTARY SCHOOLS IN CLEVELAND, OHIO, 1916 [1]

Intervals (per cent)	Number of Schools
1	2
85–89	1
80–84	24
75–79	42
70–74	23
65–69	4
60–64	2
Total	96
Mode	
Mean	
Median	

[1] Data from C. H. Judd, *Measuring the Work of the Public Schools* (Survey Committee of the Cleveland Foundation, Cleveland, Ohio, 1916).

4. ILLUSTRATIVE EXERCISES

a. What is the fashion (*mode*) among the marks which were given to ten of my sorority sisters in Education 1 to-day?

A, C, C, B, A, C, B, C, B, C

Is the curve bi-modal? Is it multi-modal?

b. Find three expressions for central tendency (averages) in Table XIII (page 76).

5. WHAT TO DO

Now, check below, each of the eight terms of which you have a reference knowledge:

arithmetic mean	mean
average	median
bi-modal curve	mode
central tendency	multi-modal distribution

Check them off the list on page 38 also, if your conscience will let you do so.

Remember that you will find *average* meaning any of the three measures of *central tendency* discussed above. The *average* may be expressed in terms of *mean*, *mode*, or *median*.

If you ever say medium for *median*, try to conceal that terminological crime from as many of your friends as possible!

CHAPTER VI

VARIATION FROM THE AVERAGE

You remember that the last chapter called attention to the necessity for some sort of terminology about a group of statistical data that one is handling, in order that it could be discussed and interpreted analytically. It is not sufficient simply to refer to an entire table or figure item by item or as a complete unit. The real significance of the related measures cannot be revealed in that way.

It has been seen that one logical way to describe a group of facts is to speak of it in terms of some measure of central tendency. This chapter will suggest another series of measures which in this case have to do with the manner in which the scores in a group scatter themselves around their central tendency. These are measures of the *variability* of the scores in the group.

1. RANGE

The simplest measure of variability, which speaks of the *deviation*, or dispersion, of measures from a mean, mode, or median has already been referred to in Chapter III. There, it was said, in describing the distribution of mental ability in a group of seven-

teen 6A pupils, that there was a *total range* of 52 units; that is, from 68 to 16. And, if the small collection of eleven measures used in Chapter V are reproduced here,

82, 76, 76, 76, 66, 55, 43, 34, 29, 27, 25,

it is seen that the range is $82 - 25 = 57$.

2. QUARTILE DEVIATION

Now, it seems natural, after noticing the extreme variation of the scores in this short row, to ask, Where is the middle half of them? If they are divided into four groups like this:

82, 76, *76*, 76, 66, *55*, 43, 34, *29*, 27, 25,

each fourth is called statistically a *quartile;* and it is seen that the middle 50 per cent falls between the scores 76 and 29, between the *third quartile* (Q_3) and the *first quartile* (Q_1). The median (55) is really the second quartile (Q_2), but it is never called that.

Statistical workers have agreed, instead of using the awkward phrase "the middle 50 per cent," to take one-half of the distance from Q_3 to Q_1 and call it the *quartile deviation,* (Q). In this array, it is

$$Q = \frac{Q_3 - Q_1}{2} = \frac{76 - 29}{2} = 23.5$$

3. AVERAGE DEVIATION

You noticed that in section 1 of this chapter the first and the last scores in the distribution were described in terms of their distance from each other, the total range. Now, every separate measure in a group may be spoken of in terms of its distance (deviation) from a central tendency. If we take the eleven quantities above, we can calculate the deviation of each figure from the median (55).

Array: 82, 76, 76, 76, 66, 55, 43, 34, 29, 27, 25
Deviations: 27, 21, 21, 21, 11, 0, 12, 21, 26, 28, 30

When the arithmetic average of these eleven deviations is found (19.82), it is an expression of the *average deviation*, or *mean deviation*. This measure of spread, or scatter, is most often taken from the mean. As the mean here is 53.55, the deviations would be

28.45, 22.45, 22.45, 22.45, 12.45, 1.45,
10.55, 19.55, 24.55, 26.55, 28.55;

and the average of them, 19.95. Therefore, 19.95 is the real average deviation.

We have, then, a third way of describing the manner in which the scores in this array deviate from the central tendency, the average deviation. Sometimes the median, instead of the mean, of the deviations is taken; and then the unit of measure is called the *median deviation*. In this problem, it is 22.45.

4. STANDARD DEVIATION

But the most accurate measure of the variability of a group of scores is known as the *standard deviation.* The theory of this is exactly the same as in the case of the average deviation or the median deviation. However, there are two differences in the method of calculation. (*a*) The standard deviation is always taken from the arithmetic average, or mean; never from any other measure of central tendency. (*b*) The sign of each separate deviation (plus or minus) is retained, but the minuses are all finally disposed of by squaring. ($-2^2 = +4$). The standard deviation (*S. D.* or *sigma*) is the square root of the mean of the squared deviations.

To illustrate with the short array above:

A. Scores
82, 76, 76, 76, 66, 55, 43, 34, 29, 27, 25

B. Deviations from the mean (D)
28.45, 22.45, 22.45, 22.45, 12.45, 1.45
— 10.55, — 19.55, — 24.55, — 26.55, — 28.55

C. Deviations squared (D^2)
809.40, 504.00, 504.00, 504.00, 155.00, 2.10,
111.30, 382.20, 602.70, 704.90, 815.10

D. Deviation squares added (ΣD^2)
5094.70

E. Mean of the squared deviation $\left(\frac{\Sigma D^2}{N}\right)$

$$\frac{5094.70}{11} = 463.15$$

F. Square root of the mean of the squared deviation $\sqrt{\frac{\Sigma D^2}{N}}$

$\sqrt{463.15} = 21.52$

A fourth unit of description, then, for the spread (deviation) of measures from the central tendency is the standard deviation (*sigma*). The Greek letter for this is σ. This is the most accurate of all, and is used most often by careful statistical workers. You will find the quartile deviation, or *semi-interquartile range*, (Q), used very often, though, in educational literature, especially in the more popular reports in the magazines.

5. PERCENTILES

When you begin to teach, you will sometimes find it desirable to rank the pupils in a class on the basis of some personality trait you have measured or in terms of their success in doing some school task you have assigned. In Table X above (page 61), you will remember that the ten different scores which your group of seventeen pupils made in an arithmetic test were put in rank order. This situation is repeated in Table XIV, but the relative rank of each individual pupil is indicated from rank 1 to rank 17. Notice that each of the pupils who made a score of 84 is given a rank of 5.5, the midpoint of their combined rank positions. Likewise, all of the five pupils who had scores of 80 are ranked 11 because the

middle of this 80 group is eleven ranks from the top of the array.

TABLE XIV

THE RELATIVE RANK OF SEVENTEEN PUPILS IN TERMS OF THEIR ACHIEVEMENT IN AN ARITHMETIC TEST

Rank	Pupils' Scores
1	2
1	95
2	94
3	90
4	85
5.5	84
5.5	84
7.5	81
7.5	81
11	80
11	80
11	80
11	80
11	80
14.5	76
14.5	76
16	70
17	65

Now, a better measure of the rank of any pupil in a group is in terms of his position in the array, with all pupils who had lower scores than his below him. If he has the highest score, his *percentile* rank is 100, as all (100 per cent) of the scores are below

his. If he has the lowest score, his percentile rank is, of course, zero (0 per cent below). If his score is median in value, his percentile rank is 50 (50th percentile).

Now, in section 2 above, two other percentile positions have already been discussed, the 25th percentile (Q_1) and the 75th percentile (Q_3). These five positions are illustrated in Table XV; and it is seen that, in this small group of seventeen pupils, he who made a score of 95 is at the 100th percentile position, the pupils who had score 84 are at the 75th percentile, score 80 indicates the 50th percentile, 76 the 25th, and 65 just the zero place, the lowest of all.

You will find in reading educational literature that the percentile positions which are most often used to indicate rank in a frequency distribution are the ten from the tenth to the 100th percentile. Table XVI gives a calculation of these for the same group of seventeen pupils, using again the arithmetic scores. The 50th percentile is, of course, the median (80). The other positions are calculated in the same manner as the median. For example, for the tenth percentile, one-tenth of the frequencies are counted from the bottom, and the cumulative figures in column 3 make this easy. Table XVII is given to show how the percentile positions come out when the distribution is grouped by class intervals.

TABLE XV

PERCENTILE RANKS FOR FIVE PUPILS IN A GROUP OF SEVENTEEN WHO TOOK AN ARITHMETIC TEST

Scores	Frequency	Five Percentile Positions
1	2	3
95	1	95=highest score (100th percentile)
94	1	
90	1	
85	1	
84	2	84=Q_3 (75th percentile)
81	2	
80	5	80=Median (50th percentile)
76	2	76=Q_1 (25th percentile)
70	1	
65	1	65=lowest score (0th percentile)
Total	17	

TABLE XVI

THE RANK OF EACH OF SEVENTEEN PUPILS IN AN ARITHMETIC TEST IN TERMS OF THE PER CENT OF SCORES LOWER THAN THEIRS

Scores	Frequency	Cumulative Frequency	Percentiles	Scores
1	2	3	4	5
95	1	17	100	95
94	1	16	90	94
90	1	15	80	85
85	1	14	70	84
84	2	13	60	81
81	2	11	50	80
80	5	9	40	80
76	2	4	30	80
70	1	2	20	76
			10	70
65	1	1	0	65
N	17			

TABLE XVII

THE PERCENTILES IN A FREQUENCY DISTRIBUTION OF MENTAL TEST SCORES

Step Intervals	Frequency	Cumulative Frequency	Percentile	Scores
1	2	3	4	5
65–69	2	17	100	68
60–64	0	15	90	66
55–59	1	15	80	55
50–54	5	14	70	53
45–49	2	9	60	51
40–44	0	7	50	49
35–39	3	7	40	40
30–34	0	4	30	37
25–29	0	4	20	24
20–24	2	4	10	19
15–19	2	2		16
N	17			

6. RELATIVE VARIABILITY

Frequently, in educational investigations, it is necessary to compare the manner in which the measures in two groups of data deviate from their respective central tendencies. This is one way usually used to express the relative variability of the two arrays. (Another way will be discussed in the next section.)

For example, you might want to know whether the boys in your class have a greater variability in life age than the girls, or whether handwriting speed

has a wider spread in the girls' group. Suppose your principal asks about the relative variability of mental ability and of arithmetic achievement in your group of seventeen 6A pupils (Table XVIII). Common sense will tell you that the measures of central tendency (mode, mean, median) say nothing about variability because they are point measures, not spread measures. And you can understand that the measures of variability themselves (range, Q, $A.D.$, σ) cannot either, because in one array the unit is mental test scores and in the other arithmetic achievement scores. It is necessary to find a common measuring scale which can be applied in both groups of data alike. This is taken to be the *coefficient of variation* (V) which is simply *sigma* divided by an average.

Columns 2 and 4 in Table XVIII show that V, the coefficient of variation, for the mental scores is 35.67, and for the arithmetic score 6.90. Evidently the spread of mental ability in this group of seventeen pupils is much wider than is the spread of arithmetic achievement in the particular test used; and this is apparent just by inspection, as the arithmetical array is only about half as long as the mental test array. The per cent of variability may be found by dividing one V by the other $\left(\frac{6.90}{35.67}\right)$. Thus it may be said that in arithmetic the pupils are only 19.3 per cent as variable as in mentality.

TABLE XVIII

THE RELATIVE VARIABILITY OF A GROUP OF SEVENTEEN 6A PUPILS IN MENTAL ABILITY AND IN ARITHMETIC ACHIEVEMENT

Mental Test Scores	Frequency	Arithmetic Achievement Scores	Frequency
1	2	3	4
65–69	2	90–95	1
60–64	0	85–89	3
55–59	1	80–84	9
50–54	5	75–79	2
45–49	2	70–74	1
40–44	0	65–69	1
35–39	3		
30–34	0		
25–29	0		
20–24	2		
15–19	2		
N	17	N	17.
Mode	52.5	Mode	82.5
Mean	43.68	Mean	81.91
Median	48.75	Median	82.50
Total Range	54	Total range	30.
Q	9.17	Q	2.36
A. D.	13.32	A. D.	3.84
Sigma (σ)	15.58	Sigma (σ)	5.65
V	35.67	V	6.90

7. OVERLAPPING

A final way to compare two groups of related data is in terms of the *overlapping* of their distributions. Suppose you have your room of twenty-two 6B and seventeen 6A pupils read the same material silently and ask them to read as rapidly as they can but thoughtfully, so that they may remember the

details of the story. Table XIX shows what the results might be. The lower group has read at an average rate of 231.43 words per minute; the higher at a rate of 283.33 words.

TABLE XIX

DISTRIBUTION OF THE NUMBER OF WORDS READ PER MINUTE BY 22 6B AND SEVENTEEN 6A PUPILS WHEN THEY ARE ASKED TO READ SILENTLY, AS RAPIDLY AS POSSIBLE, BUT THOUGHTFULLY

Scores	6 A	6 B
1	2	3
360–379	2	
340–359	1	
320–339	2	1
300–319	1	2
280–299	3	2
260–279	6	1
240–259	0	2
220–239	0	7
200–219	0	2
180–199	0	2
160–179	2	1
140–159		2
Total	17	22
Median	283.33	231.43

Now, overlapping is generally measured in terms of the per cent of one group which reaches or exceeds the median of the other. Just a glance at the two arrays (columns 2 and 3) shows that about four or five of the twenty-two 6B pupils are above the median 6A score (283.33). This would be about a fifth of them, and after careful calculation it turns

out that exactly 21.2 per cent of the 6B group reaches or exceeds the median of the 6A group. This relative difference in the spread and in position of the two distributions is pictured graphically in Figure 17. Count the areas representing 6B pupils to the right of the 6A median. You see that there are probably about 4.5 of them. Here is a very definite and illuminating way of comparing groups of data like this, and you will find educational students using it often.

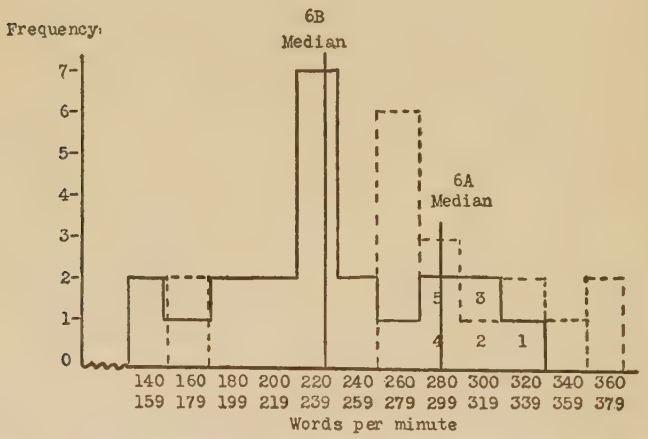


FIG. 17.—THE OVERLAPPING OF 6B AND 6A PUPILS IN SPEED OF SILENT THOUGHTFUL READING.

8. ILLUSTRATIVE EXERCISES

a. My sixth-grade arithmetic class of forty children in the training school worked on a list of

twenty simple examples for ten minutes to-day, and then I let them appoint a committee to score the results. I told them to report class results only. They found the median to be ....., the total range ....., and Q ..... Here are the individual scores:

20, 19, 10, 16, 15, 14, 17, 19, 20, 20, 11, 10, 13, 13, 17, 16, 15, 15, 20, 6, 18, 12, 11, 19, 15, 13, 12, 16, 15, 14, 14, 11, 12, 20, 9, 20, 16, 15, 14, 13

b. We were discussing the new junior-college movement in Education 1 to-day, and the question of the proper size of student body came up. Some thought a total of fifty students would be enough to insure good work. Some were sure there ought to be at least two hundred. To close the discussion our instructor reported the actual enrollment in 38 junior colleges (1927) to be as follows:

135, 160, 163, 163, 254, 57, 328, 116, 600, 363, 28, 383, 285, 171, 634, 94, 479, 229, 100, 152, 213, 83, 132, 250, 49, 159, 762, 385, 1425, 366, 1143, 267, 113, 115, 237, 260, 144, 174

This array is fine to use for demonstration of facts of variability. Find the difference between (*a*) the average deviation and (*b*) the standard deviation.

c. My percentile rank in the true-false test yesterday in educational psychology was 70. The in-

structor put the entire list of scores on the board and computed the position of all of the ten-percentiles from the 0th to the 100th inclusive. Here are the 38 scores. Arrange them as in Table XVII (page 86). One hundred and fifty was the highest possible score.

130, 135, 150, 100, 90, 120, 125, 140, 145, 145, 143, 144, 147, 148, 139, 142, 141, 138, 100, 100, 95, 128, 127, 143, 129, 137, 139, 141, 140, 149, 143, 141, 139, 135, 121, 128, 140, 147

d. Our instructor in educational psychology says that, if all men and all women be considered, there are greater differences in native intelligence among the men as a group than among the women. He says that this means that in the men's group there are more geniuses and more morons and imbeciles than there are in the women's group. That is, there is a greater variability in mental ability among men. I know that our educational psychology class is too small a group to make the results of an intelligence test generally significant, but I thought it would be interesting to try it out anyway. The instructor said he would give me a list of the intelligence scores of the class which included twenty men and thirty-five women. The men had been tested with the Thorndike Intelligence Examination for High-School Graduates, and the women with the Thurstone Psychological Examination for College Freshmen. Here they are:

Men's Scores: (Thorndike)	99, 32, 81, 31, 27, 19, 52, 64, 66, 32 17, 44, 49, 50, 44, 67, 47, 45, 31, 51
Women's Scores: (Thurstone)	23, 58, 75, 18, 19, 10, 73, 67, 42, 25 34, 56, 6, 16, 15, 85, 55, 48, 65, 54 43, 51, 17, 45, 38, 37, 8, 83, 70, 12 13, 61, 60, 16, 90

I found *V* for the men to be . . . . . , and *V* for the women . . . . . This made the per cent of variability . . . . . That is, the women's (men's) group was only . . . . . per cent as variable in native intelligence as the men's (women's).

e. We were discussing in sociology class the other day the fact that teachers are not very keen in business; that after all, if they are sincere and honest, they must give their time and interest almost wholly to their profession; that careful saving is required if economic advancement is desired, and that too many teachers save too little systematically. I ran across some very interesting figures on amounts actually saved by Illinois teachers that night and reported them in class the next day in the form shown in Table XX (page 94).

The class was very much interested to see that there was an overlapping of . . . . . per cent; that is, that . . . . . per cent of the men (women) reached or exceeded the median of the women (men) in amount saved.

TABLE XX

DISTRIBUTION OF 1,837 ILLINOIS PUBLIC-SCHOOL TEACHERS IN TERMS OF AMOUNTS SAVED FROM THEIR ANNUAL SALARIES DURING EACH OF THE LAST FIVE YEARS, 1917 [1]

AMOUNT SAVED	FREQUENCY	
	Men	Women
1	2	3
$2000–2099	3	3
1900–1999	0	0
1800–1899	0	0
1700–1799	0	0
1600–1699	0	0
1500–1599	2	0
1400–1499	1	1
1300–1399	0	0
1200–1299	2	2
1100–1199	0	4
1000–1099	0	0
900– 999	11	12
800– 899	0	0
700– 799	2	8
600– 699	2	8
500– 599	7	19
400– 499	23	49
300– 399	23	52
200– 299	54	156
100– 199	68	447
0– 99	81	797
Total	279	1558
Median		

[1] Figures from L. D. Coffman, *Illinois School Survey* (Illinois State Teachers Association, 1917).

9. WHAT TO DO

Now, there are fourteen new words used in this long chapter:

average or mean deviation	quartile
coéfficient of variation	quartile deviation
deviation	semi-interquartile range
first quartile	standard deviation
median deviation	third quartile
overlapping	total range
percentile	variability

Check each of them that has been added to your vocabulary in educational statistics; and check them off the list on page 38, if your conscience permits. Remember that they all tell something about the way in which measures spread up and down the array from a central tendency, that percentiles give the position of individual scores in the column, and that the coefficient of variation and per cent of overlapping are ways of comparing the relative position and spread of distributions.

If you have a good reference knowledge of the thirty-seven new terms dealt with in the five preceding chapters, you are getting to be quite an educational statistician! Don't be satisfied until you *do* have some rather definite idea of what they mean, as you meet them in your required reading.

CHAPTER VII

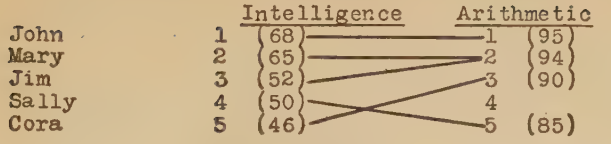
RELATIONSHIPS

Very probably you remember that your sixth-grade teacher used to talk about correlating geography with history. She meant that she wanted the content of each of these subjects to contribute to interest in and understanding of the other. As you do the required readings in your education courses, you will frequently come across the word *correlation;* and there it will most often mean an expression of *relationship* between factors, or variables, as measured in a group of pupils, for example.

1. CORRELATION

It might be that in studying the speed and the quality of their handwriting, you would discover that as speed increases the quality gets better. Or it might be you have measures of the intelligence and of the arithmetic achievement of the same pupils; and on the whole you would find that the most intelligent pupils did best in arithmetic.

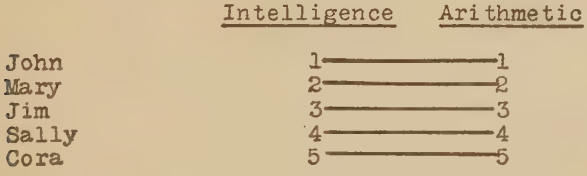
Suppose that five of these pupils are taken and arranged like this in terms of both intelligence and arithmetic scores.



Mixed Correlation

The connecting lines show the relationship, or correlation, of the scores of these five pupils. In the case of John and Mary, there is a perfect correlation, as John is highest in both intelligence and arithmetic, while Mary has second rank in both. But Jim is third in intelligence and second in arithmetic, having the same score as Mary in the latter subject. Sally is fourth in one and fifth in the other, and Cora is fifth and third. After the first two cases the relationship is mixed. It is not perfect correlation.

If the correlation were perfect and positive, the relationship of the scores of these five pupils would be like this, and the *coefficient of correlation* would be + 1.00.

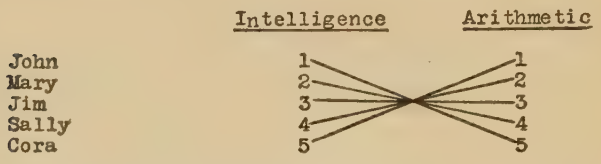


R = + 1.00

Perfect Positive Correlation

If the correlation were perfect but negative, John would be highest in intelligence and lowest in arith-

metic, Mary second highest in intelligence and second from the bottom of the ranks in arithmetic, etc. As intelligence decreases, arithmetic scores increase. The coefficient of correlation would be —1.00.



R = -1.00

PERFECT NEGATIVE CORRELATION

But one never finds either perfect positive or perfect negative correlation among the variables of a social science like education. The relationship always ranges somewhere between these two extremes; and is sometimes zero—no relationship whatever.

When the actual correlation of intelligence and arithmetic scores in the case of this small group of five pupils is calculated, it turns out to be + .75 (plus point seven five). This is positive and rather high. Correlation is said to be high when R gets close to + .80 or +.90.

R	*R*	*R*
— 1.00	0	+ .75 + 1.00

In Figure 18, these three types of relationships are represented graphically in scatter diagrams. Notice that: (*a*) for R = + 1.00, the arithmetic

ranks decrease as the intelligence ranks decrease; (*b*) for R = — 1.00, the arithmetic ranks increase as the intelligence ranks decrease; (*c*) and, for R = + .75, there is a mixture of ranks after the first two cases, which decrease together.

When the mental and arithmetic scores of all of the seventeen pupils in your 6A group are arranged to show these correlating relationships, they appear as in Table XXI. Notice that, while pupil 1 has the highest scores in both variables (columns 4 and 5) and pupil 2 has the next highest in both (making perfect positive correlation that far), pupil 3 has third highest in arithmetic but eighth in mentality, and pupil 4 has the lowest score in intelligence although he is fourth in arithmetic. After the very beginning of the arrays, there is a mixture of ranks so that, although the guess is that the correlation is positive, as the rank numbers in columns 4 and 5 seem on the whole to decrease in value from top to bottom, nothing sure can be said about how close the relationship is.

A picture of this relationship is given in Figure 19. You see that, beginning at the lower left hand corner, one pupil is lowest in both intelligence and arithmetic, one is between 20 and 24 in intelligence and between 70 and 74 in arithmetic, two are 35 to 39 intelligence and 80 to 84 arithmetic, up to the two at the extreme upper right hand corner who are highest in both variables. On the whole, there is a general trend of the scatter from lower left to upper right

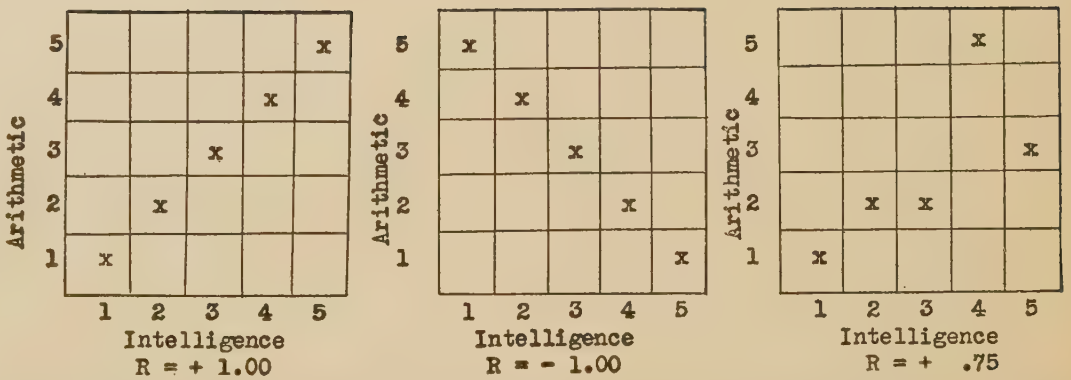


FIG. 18.—PERFECT POSITIVE CORRELATION ($R = +1.00$), PERFECT NEGATIVE CORRELATION ($R = -1.00$), AND RATHER HIGH POSITIVE CORRELATION ($R = +.75$) ILLUSTRATED GRAPHICALLY IN THREE SCATTER DIAGRAMS, OR "CORRELATION TABLES."

hand; but all paired frequencies are not in a perfectly straight line as in the first part of Figure 18.

TABLE XXI

MENTAL SCORES AND ARITHMETIC ACHIEVEMENT OF SEVENTEEN 6A PUPILS AND THE CALCULATION OF THEIR CORRELATION, ZERO ORDER

PUPILS	ARITHMETIC SCORES	MENTAL TEST SCORES	RANK ORDER		GAINS IN RANK (5 over 4)
			Arithmetic Scores	Mental Test Scores	
1	2	3	4	5	6
1	95	68	1	1	0.0
2	94	65	2	2	0.0
3	90	52	3	8	0.0
4	85	16	4	16.5	0.0
5	84	58	5.5	3	2.5
6	84	53	5.5	6.5	0.0
7	81	53	7.5	6.5	1.0
8	81	47	7.5	9	0.0
9	80	37	11	12	0.0
10	80	54	11	4.5	6.5
11	80	54	11	4.5	6.5
12	80	37	11	12	0.0
13	80	21	11	14.5	0.0
14	76	37	14.5	12	2.5
15	76	46	14.5	10	4.5
16	70	21	16	14.5	1.5
17	65	16	17	16.5	0.5
SUM					25.5

$$R = 1 - \frac{6 \Sigma G}{N^3 - 1} = 1 - \frac{6 \times 25.5}{17^3 - 1} = 1 - \frac{153}{488} = +.699 \pm .087$$

There are a number of sports outside, and in particular the two toward the bottom of the table. How-

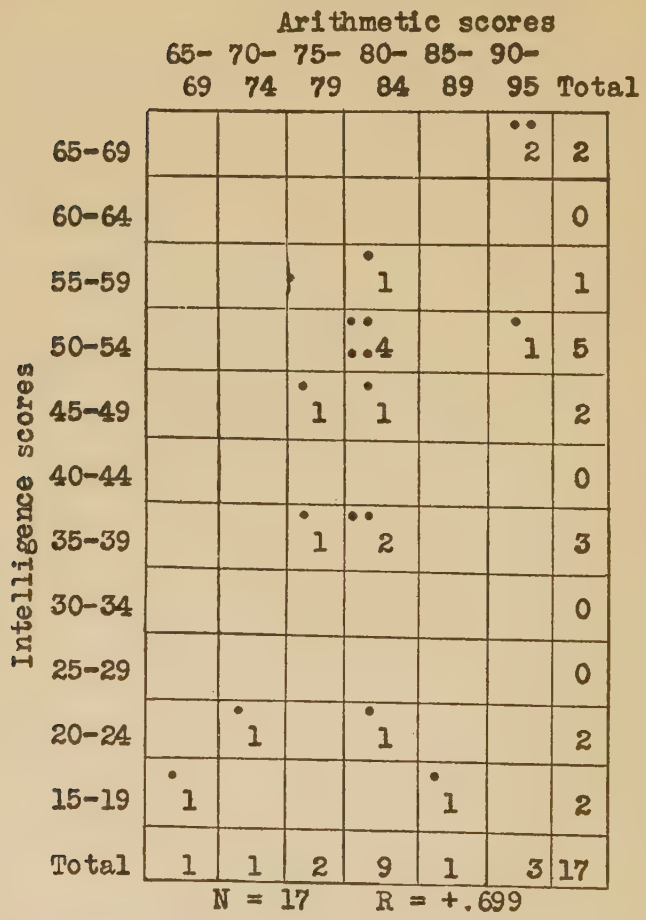


FIG. 19.—SCATTER DIAGRAM OR "CORRELATION TABLE," REPRESENTING THE RELATIONSHIP (CORRELATION) OF INTELLIGENCE SCORES AND ARITHMETIC ACHIEVEMENT SCORES IN A 6A CLASS OF SEVENTEEN PUPILS.

ever, with these exceptions the scatter shows rather a straight line relationship, and without calculation at all a good guess can be made that the correlation is positive and rather high.

In Table XXI, page 101, the details of the calculation are given. What is known as a rank difference method is used because the arrays are so short (only seventeen cases). The coefficient of correlation proves to be + .699. This is positive and rather high. There is a *probable error* of plus or minus .087, and this means that the coefficient may be as large as .786 or as low as .612.

2. PARTIAL AND MULTIPLE CORRELATION

Now, when you begin to teach you will understand that, in dealing with a situation such as that above, it is not very sensible to assume that the relationship of two variables such as mentality and arithmetic achievement are entirely independent and unaffected by other factors present. The fact is that there are a number of other things which no doubt influenced these scores on the days when the tests were given. For example, pupil 4 had the lowest score in intelligence, and that may have been because he had trouble in reading parts of the intelligence-test booklets. Pupil 17 may have had a low score in arithmetic because she was unwell when the test was given. Pupil 16 may be too young in life age for success in the group. Pupil 13 may be a slow writer, and so he scored low in both tests. And there very

probably were dozens of other factors which had some influence on the relationship of the two variables measured.

When none of them are taken account of in the calculation as in the example above, the coefficient of correlation is said to be *order zero*, and really should be written, $R_{12 \cdot 0} = +.699$ (the correlation between variables 1 and 2 with no other related factors accounted for). If one other factor is measured and included in the calculation, the coefficient would be of the *first order*, and would be written $R_{12 \cdot 3}$. This is known as a coefficient of *partial correlation*. Of course, as many factors ought to be accounted for in any investigation as there are skill, funds, and time to handle. The more included, the closer one comes to the true relationship.

Frequently, you will find research workers reporting the relationship of a number of variables as a group to a *criterion* which they affect as a combination of influences acting as a unit. This is known as *multiple correlation* and is written $R_{1 \cdot 234}$ or $R_{1(234)}$, if there are three independent variables.

Further, by the use of path coefficients and coefficients of determination, one may express the relative degrees of causal relationship of independent variables to a criterion in terms of per cent. So, cheer up! You can understand per cents; and when more educational writers learn how to use coefficients of determination your reading will be easier and more understandable.

Now, it will be impossible for you to understand all of the technic and the implications of the method of partial and multiple correlation until you take your formal course in educational statistics. But try to get a good reading knowledge of the terms commonly found in reports where this method is employed. One of the most important uses of this technic is in *prognosis;* that is, in predicting the success of an individual on the basis of a study of the relationship of a number of factors to a criterion.

For example, if the fifth-grade teacher had sent to you with the group of twenty-two pupils now your 6B's, a true statement of the prediction value, or weight, of each of several important factors in their school success (*the regression equation*), you would be better able to arrange their 6B work so as to insure the best future success on the part of each individual pupil. This regression equation might look like this:

School success = 60 Intelligence + 15 Attendance + 8 Home influence + 2 Physical development + 37.

For each incoming pupil, you would substitute for intelligence, attendance, home influence, and physique, numerical values resulting from investigation of these factors in his case. Then, when you had added the five members on the right side of the equation, you would have an expression of amount for his probable school success in 6B.

3. ILLUSTRATIVE EXERCISES

a. We had a discussion in Education 1 about intelligence tests, and whether they *really* measured intelligence, and whether success in high school, for example, depended upon the kind of intelligence the usual tests measured. I remembered that, as a high-school senior, I had taken Thorndike's Intelligence Examination for High-School Graduates; and I told our instructor that I thought I could get my high-school principal to send me the scores for the entire class and all their high-school marks also. It was a small school with only twenty-five graduating last year.

After the data had come, I averaged the subject marks for each pupil and made two arrays for the two variables I had which appeared like columns 2 and 3 in Table XXII.

How did my report to the class look when all the columns were filled out and all the computations made? The coefficient of correlation I found was ..... What did this mean about the relationship between the native intelligence and the high-school success of these twenty-five high-school pupils? Did I find the *true* relationship between these two variables? *No* (*Yes*) How many and what other factors should I include in a partial or multiple correlation problem to come near to a discovery of the *real* relationship between intelligence and high-school work?

TABLE XXII

THE CORRELATION OF (2) NATIVE INTELLIGENCE AS MEASURED BY MEANS OF THE THORNDIKE INTELLIGENCE EXAMINATION FOR HIGH-SCHOOL GRADUATES AND (3) HIGH-SCHOOL RECORD IN TERMS OF THE AVERAGE OF ALL SUBJECT MARKS FOR FOUR YEARS IN A GROUP OF 25 HIGH-SCHOOL GRADUATES [1]

PUPILS	INTELLIGENCE	HIGH-SCHOOL RECORD	RANK ORDER		GAINS IN RANK (5 over 4)
			Intelligence Scores	High-School Marks	
1	2	3	4	5	6
1	58.9	82.0			
2	58.0	86.0			
3	83.9	83.3			
4	43.6	84.3			
5	83.6	85.1			
6	71.4	90.9			
7	65.0	81.8			
8	60.4	87.0			
9	78.7	85.7			
10	66.1	87.1			
11	72.7	85.8			
12	80.0	89.4			
13	73.0	83.9			
14	68.9	84.9			
15	42.1	70.3			
16	40.5	84.9			
17	83.2	91.2			
18	97.9	94.7			
19	55.3	89.8			
20	42.5	79.6			
21	43.2	86.6			
22	72.2	86.2			
23	54.3	81.8			
24	57.8	77.3			
25	46.3	72.6			
Sum					

$$R = 1 - \frac{6 \Sigma G}{N^2 - 1} =$$

[1] F. L. Whitney, *The Prediction of Teaching Success*, Journal of Educational Research Monograph Series, Number 6 (Public School Publishing Co., Bloomington, Illinois, 1924).

b. The Prediction of Arithmetic Success.[4] I remember that, when I was in the fifth grade, I had to stay out of school almost all of the second semester with influenza, and when I went back in the fall I was put in what they called a Coach Room with about two dozen other children who were behind in their work. We had an especially good teacher, and nearly all of the group made rather rapid progress, so that when we were ready to be sent to the 6B rooms we were just about "up with the grade."

Our Coach Room teacher sent with each pupil on the back of his report card a regression equation for each of the principal subjects he had been coached in. That for mathematics looked like this:

Arithmetic success = 75 Intelligence + 10 Home influence + 60 Reading ability + 70 Reasoning ability + 40 Ability in the four fundamental operations + 50.

This equation was used by the new teacher of each pupil, and it gave her an analytical notion of his probable success in 6B arithmetic work. For example, when substitution had been made with the scores for the five factors included, my regression equation looked like this:

Arithmetic success = (75 × 80) Intelligence + (10 × 500) Home influence + (60 × 72) Reading ability + (70 × 5) Reasoning ability + (40 × 70) Ability in the four fundamental operations + 50.

[4] Suggestion and data from F. L. Whitney, *The Growth of Teachers in Service* (Century Co., 1927).

This showed my teacher that, although my chances for success in arithmetic in the 6B class were rather good owing to a fair intelligence score and good reading ability, I needed especial help with reasoning problems and in drill on addition, subtraction, multiplication, and division.

Compare this prediction of arithmetic success with that of my friend, Mary, whose measures in the same five factors were as follows:

Intelligence	70
Home	500
Reading	70
Reasoning	6
Fundamental operations	75

Although Mary's intelligence score was lower, her reading ability was about the same as mine, and her specific arithmetic abilities some higher. Were her chances for success in 6B arithmetic work on the whole greater (less) than mine?

4. WHAT TO DO

Now, with these final eleven statistical words below, you have a total of 62 used in this little book:

coefficient of correlation
correlation
criterion
first order correlation
multiple correlation
partial correlation
probable error
prognosis
relationship
regression equation
zero order correlation

Can you with a clear conscience check them all off the list on page 38? They are all given in alphabetical order in the glossary, which comes next.

It would be a good review and test to cover up the definitions in this glossary and try to tell your roommate what each term, *arithmetic mean* to *zero order correlation*, means. These definitions are rather stilted, though; and the best way to review their meaning will be to look up each which is not perfectly clear on the page of the textbook indicated in parenthesis.

GLOSSARY

The page numbers after each word indicate the place of its first use in the textbook.

1. *Arithmetic mean:* The total of several measures divided by their number; Arithmetic average; *Mean* (*M*) (p. 72).
2. *Array:* A group of measures or scores arranged in decreasing serial order according to size (p. 54).
3. *Attribute:* A trait or quality either absent or present (p. 51).
4. *Average:* A general term which gives as a single figure the typical value of a group of figures; *Central tendency* (*A*) (p. 71).
5. *Average deviation:* The arithmetic average, or mean, of all the deviations from a central tendency, usually the average; *Mean deviation* (*AD, MD*) (p. 80).
6. *Bi-modal curve or distribution:* A distribution having two modes (p. 72).
7. *Central tendency:* The typical value of a group of measures; *Average* (p. 71).
8. *Chart:* A graphical representation of a group of related statistical facts; *Graph; diagram; figure* (p. 59).
9. *Classification:* The arrangement of the original data of an investigation on succeeding levels of generalization (p. 51).
10. *Class interval:* A small group of measures used in the tabulation of the larger, total group; *Step interval* (p. 55).

11. *Coefficient of correlation:* An expression of the relationship between two sets of measures of traits, abilities, scores, etc., of individuals or of groups (*r, R, rho*) (p. 97).
12. *Coefficient of variation:* An expression of the relative variability of two or more groups of data; The standard deviation divided by an average (*V*) (p. 87).
13. *Correlation:* The extent to which increases in one thing measured are accompanied by increases or decreases in another related thing measured; *Relationship* (p. 96).
14. *Criterion:* The factor, or variable (dependent variable), whose value is sought in a study of relationships (p. 104).
15. *Deviation:* The dispersion of measures around a central tendency; The difference in value of the measures in a distribution in relation to an average (p. 78).
16. *Diagram:* A graphical representation of a group of related statistical facts; *Graph; figure; chart* (p. 59).
17. *Distribution:* An arrangement of measures in the order of their value and frequency (p. 55).
18. *Factor:* A trait, ability, or measure considered in a study of investigation; *Variable* (p. 40).
19. *Figure:* A graphical representation of a group of related statistical facts; *Graph; diagram; chart* (p. 59).
20. *First order correlation:* The relationship of two variables when the influence of one other factor in the same realm of influence is held constant ($r_{12.3}$); Second order is $r_{12.34}$; Third order is $r_{12.345}$, etc. (p. 104).
21. *First quartile:* One-fourth the distance up the distribu-

tion from the smallest measure to the largest; the 25th percentile (Q_1) (p. 79).

22. *Frequency:* The number of times a score or other measure is found in a group of data (F, f) (p. 40).
23. *Function:* "Any measurable trait which any reputable investigator has studied" (p. 39).
24. *Graph:* A graphical representation of a group of related statistical facts; *Figure; diagram; chart* (p. 59).
25. *Investigation:* A study of data or measures obtained in a specific situation; *Study* (p. 40).
26. *Mean:* The total of several measures divided by their number; *Arithmetic mean* (M) (p. 72).
27. *Median:* The middle score or measure when measures are arranged according to size; The midpoint in a range of scores having the same number of measures on either side (*Mdn., Med.*) (p. 72).
28. *Median deviation:* The median of all deviations from a central tendency (*Med. Dev.*) (p. 80).
29. *Mode:* The most frequent figure or size of measure; The "most fashionable" measure (*Mo.*) (p. 72).
30. *Multiple correlation:* The combined relationship of several independent related factors as a group to a dependent variable (p. 104).
31. *Multi-modal distribution or curve:* A curve having more than one mode (p. 72).
32. *Normal curve:* A distribution in which the same number of measures, decreasing in number equally toward the smallest and the largest, are found on either side of the central tendency; *Normal distribution* (p. 65).
33. *Normal distribution:* A distribution in which the same number of measures, decreasing in number equally toward the smallest and the largest, are found on

either side of the central tendency; *Normal curve* (p. 65).

34. *Organization:* The first level of arrangement of the data of a study (p. 53).
35. *Overlapping:* The per cent of one group which reaches or exceeds the median of the other (p. 88).
36. *Partial correlation:* The relationship of two variable factors when the influence of other measured variables in the same realm of influence is held constant ($r_{12.34 \ldots \ldots n}$) (p. 104).
37. *Percentile:* A point on a distribution below which a given proportion of cases are found; For example, the median is the 50th percentile, Q_1 is the 25th percentile, Q_3 is the 75th percentile (p. 83).
38. *Population:* All individuals or cases of one kind in existence (p. 40).
39. *Probable error:* A measure of the variability of an obtained measure from a true measure (*P.E.*) (p. 103).
40. *Prognosis:* The mathematical process of predicting the value of a dependent factor as influenced by several related independent variables (p. 105).
41. *Quartile:* One-fourth of the total range of all the measures in a distribution (p. 79).
42. *Quartile deviation:* Half the difference between the first and the third quartiles in a distribution of measures; *Semi-interquartile range* (*Q*) (p. 79).
43. *Question list:* A formal inquiry consisting of lists of questions arranged in different ways asking for information needed in a specific investigation; *Questionary; Questionnaire* (p. 42).
44. *Questionary:* A formal inquiry consisting of lists of questions arranged in different ways asking for information needed in a specific investigation; *Question list; Questionnaire* (p. 41).

45. *Questionnaire:* A formal inquiry consisting of lists of questions arranged in different ways asking for information needed in a specific investigation; *Question list; Questionary* (p. 41).
46. *Range:* The difference between the largest and the smallest measures in a distribution; *Total range* (p. 53).
47. *Rank:* The order of measures or scores when listed in an array from largest to smallest (p. 54).
48. *Regression equation:* An expression of the true predictive value, or weight (based on partial correlation), of several independent factors as related to a dependent variable, the criterion (p. 105).
49. *Relationship:* The degree of correspondence or difference between two measurements of traits or abilities in individuals or groups; *Correlation* (p. 96).
50. *Sampling:* The number of measures of an entire population obtained in an investigation (N) (p. 40).
51. *Selected group:* A group which is not representative of a larger sampling or of the entire population dealt with (p. 40).
52. *Semi-interquartile range:* Half the difference between the first and the third quartile; *Quartile deviation* (Q) (p. 82).
53. *Skewness:* The degree of difference between a normal curve and a curve flattened to the left (negative skewness) or to the right (positive skewness) (p. 68).
54. *Standard deviation:* The square root of the average of the squares of all deviations from the mean; The root-mean-square deviation; *Sigma* (*S.D.*, σ) (p. 81).
55. *Step interval:* A small group of measures used in the tabulation of the larger, total group; *Class interval* (p. 55).

56. *Study:* An investigation of measurable facts in a chosen field or situation; *Investigation* (p. 39).
57. *Table:* A group of numerical data arranged, or classified, by frequency in class intervals (p. 55).
58. *Third quartile:* Three-fourths the distance up the distribution from the smallest measure to the largest; The 75th percentile (Q_3) (p. 79).
59. *Trait:* A factor or function used as a unit of measure in a study or investigation; *Variable* (p. 39).
60. *Variable:* A trait, ability, or other factor which *varies* in relation to other influences present; A factor existing in different amounts at different times under different conditions (p. 51).
61. *Variability:* The relative size of each measure in a group as compared with the central tendency (p. 78).
62. *Zero order correlation:* The relationship of two variables without taking account of the possible influence of other factors in the same realm of influence (r_{12} or $r_{12.0}$) (p. 104).

FORMULAS FREQUENTLY USED

I. $$\text{Arithmetic mean} = \frac{\text{Sum of the measures}}{N}$$

$$A = \frac{\Sigma\ (\text{Measures})}{N}$$

II. $$\text{Median} = \text{Lower limit of middle class} + \frac{\frac{N}{2} - \text{Partial sum}}{\text{Class frequency}} \times \text{Class interval}$$

$$\text{Med.} = LL + \frac{\frac{N}{2} - \text{S}}{F} \times \text{I}$$

$$\text{Median} = \text{Upper limit of middle class} - \frac{\frac{N}{2} - \text{Partial sum}}{\text{Class frequency}} \times \text{Class interval}$$

$$\text{Med.} = UL - \frac{\frac{N}{2} - \text{S}}{F} \times \text{I}$$

III. $$\text{Average deviation} = \frac{\text{Arithmetical sum of deviations from the average}}{N}$$

$$A.D. = \frac{\Sigma D}{N}$$

IV. $$\text{Standard deviation} = \sqrt{\frac{\text{Sum of squares of deviations}}{N}}$$

$$\text{Sigma} = \sqrt{\frac{\Sigma D^2}{N}}$$

V. $$\text{Coefficient of variation} = \frac{100 \times \text{Standard deviation}}{\text{Average}}$$

$$V = \frac{100\,\sigma}{A}$$

VI. $$\text{Skewness} = \frac{3\ (\text{Arithmetic mean} - \text{median})}{\text{Standard deviation}}$$

$$\text{Sk.} = \frac{3\ (A - \text{Med.})}{S.\ D.}$$

VII. $r_{12} = \dfrac{\text{Sum of the deviations of Variable 1} \times \text{Variable 2}}{\text{Square root of the product of the sum of the deviations of Variable 1 squared} \times \text{those of Variable 2 squared}}$

$$r = \frac{\Sigma\, x \,.\, y}{\sqrt{\Sigma\, x^2 \,.\, \Sigma\, y^2}}$$

VIII. $\text{rho} = 1 - \dfrac{6 \times \text{Sum of the differences between ranks squared}}{N(N^2 - 1)}$

$$\rho = 1 - \frac{6\,\Sigma\, D^2}{N(N^2 - 1)}$$

IX. $R_{12} = 1 - \dfrac{6 \times \text{Sum of the gains in rank order of Variable 1 over Variable 2}}{N^2 - 1}$

$$R_{12} = 1 - \frac{6\,\Sigma\, G}{N^2 - 1}$$

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This list includes a few of the most recently published textbooks in educational statistics, one of which you will no doubt use when you take your formal course later in your teachers-college experience.

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